Essays in Dynamic Macroeconomics and Public Finance

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The Faculty of Economics, Business Administration and Information Technology of the University of Zurich hereby authorizes the printing of this dissertation, without indicating an opinion of the views expressed in the work.

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Chairman of the Doctoral Board: Prof. Dr. Dieter Pfaff

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Gancia, Gino, Andreas Müller, and Fabrizio Zilibotti (2013). "Structural Development Accounting," in *Advances in Economics and Econometrics: Theory and Applications, Thenth World Congress*, D. Acemoglu, M. Arellano, and E. Dekel (eds.), Cambridge, UK: Cambridge University Press.

Chapter 1

Structural Development Accounting

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1.1 Introduction

New technologies do not diffuse instantaneously, and adoption lags are considered a major determinant of productivity differences across firms and nations. In a classic paper, Griliches (1957) documents that new more productive seeds of hybrid corn diffused slowly across US agricultural regions, with a 15-year lag between adoption in Iowa and Alabama, and that their diffusion was affected by local conditions, such as geography and market potential. The spread of more recent technologies follows a similar pattern. Kiessling (2009) reports evidence of slow adoption of information and communication technology diffusion both between and within countries. For instance, while personal computers became available in the early 1980s, in 2006 the percentage of the population using computers amounted to 80.6% in US, 36.3% in Spain, 5.6% in China and 2.7% in India. Cross-country studies confirm that technology adoption depends both on country-specific factors and on characteristics of new technologies. For example, a McKinsey (2001) report on India identifies as a major source of inefficiency the fact that firms are too small to benefit from the best technologies and that these may require skills that the country does not possess. The importance of local economic conditions is also stressed by Caselli and Wilson (2004), who show that countries import technologies complementing their abundant factors, and by Ciccone and Papaioannou (2009), who find that human capital fosters the adoption of skill-augmenting technologies. At the aggregate level, there is evidence that differences in technology are a key determinant of cross-country income disparities. A large body of research measuring total factor productivity (TFP) as the Solow residual of an aggregate production function typically finds the latter to account for roughly 50% of observed differences in output per worker. Beyond being a measure of our ignorance, this residual is nothing but a generic notion of technology, i.e., the mapping from factors to aggregate production.

What all these pieces of evidence suggest is that, if we are to understand income disparities, we need a theory for how different types of technologies are developed and adopted across countries. In turn, this requires unbundling the concept of TFP into a set of heterogeneous technologies and to identify what country-specific factors facilitate the adoption of certain innovations more than others. To this end, a parsimonious description of technology is provided by the following aggregate production function:

$$Y = K^{\alpha}\left(\left[(A_L L)^{\frac{\upsilon-1}{\upsilon}} + (A_H H)^{\frac{\upsilon-1}{\upsilon}}\right]^{\frac{\upsilon}{\upsilon-1}}\right)^{1-\alpha}, \tag{1.1}$$

where Y, K, H and L are output, physical capital, skilled and unskilled labor, respectively. The state of technology is identified by the parameters A_L and A_H, which measure the efficiency with which the economy uses unskilled and skilled labor, respectively. The parameter υ, instead, captures the elasticity of substitution between the two types of workers. Given data on factors and a value for υ, any differences in Y can be generated by allowing technology, A_L and A_H, to vary. While accounting exercises based on (1.1) are certainly useful, the crucial question is to understand *how* technologies are developed and *why* they may differ across countries. Providing a theoretical answer to these questions and confronting it to the data is the main goal of this paper.

Building on Acemoglu and Zilibotti (2001) (henceforth, AZ01) and Gancia and Zilibotti (2009), we propose a theory of directed technical change and technology adoption that yields a micro-founded version of the aggregate production function (1.1). In the model, an advanced economy, identified with the US and called for simplicity the North, develops endogenously the world technology frontier, represented by the pair (A_{LN}, A_{HN}). As in models of horizontal innovation, the world technology frontier is given by the stock of existing machines and, as in models of directed technical change, R&D effort can be devoted to develop H- or L-complement machines.[1] In the benchmark case, we assume that there is no trade in technology – e.g., due to the lack of international protection of intellectual property rights (IPR) – so that new machines are sold in the North only.[2] As a result, the equilibrium skill bias of the world technology frontier is proportional to the skill endowment of the North. To capture the advantage of backwardness emphasized, among others, by Nelson and Phelps (1966), and Acemoglu, Aghion and Zilibotti (2006) we assume that all other countries can *adopt* existing technologies at a cost which is decreasing in their distance from the frontier. Besides this cost, technology adoption - just like innovation - is profit-driven and depends on local economic conditions, such as the abundance of complementary factors (K, L and H) and the size of domestic markets. This combination of the theory of directed technical change with international knowledge spillovers allows us to build a tractable model of cross-country technology differences suitable for quantitative analysis.

The resulting model yields structural equations that can be used to estimate its two key parameters: the elasticity of substitution between the skilled and unskilled labor, ϵ, and the elasticity of the adoption cost to the technology gap, ξ, capturing exogenous barriers to knowledge flows. From these estimates, our methodology allows us to tease out the relative importance of two distinct sources of low productivity: technology inappropriateness and

[1] Models of horizontal innovation are surveyed in Gancia and Zilibotti (2005). The notion of directed technical change stretches back to Kennedy (1964). Acemoglu (1998) constructs a quality-ladder model of directed technical change to study the patterns of wage inequality in the US. AZ01 proposes the first model of directed technical change with horizontal innovation and apply it to the study of cross-country productivity differences (see also Acemoglu 2002).

[2] We relax this assumption in an extension where we introduce international license contracts on the use of technology.

distance to frontier. To see why, note that when barriers to adoption are very low, a country will operate with the best technologies; yet, to the extent that frontier technologies are highly skill biased they will be of limited use in skill-scarce countries, thereby generating low aggregate productivity. On the contrary, countries well inside the frontier are free to choose a more optimal mix of technologies, so that their low productivity will be mostly explained by barriers to adoption, rather than the skill-technology mismatch.

To estimate the elasticity of substitution between the skilled and unskilled labor, we use time-series data on the skill premium and the relative skill supply in the US (the frontier economy). The second parameter, ξ, measuring barriers to technology adoption, is instead estimated from a micro-founded version of equation (1.1). That is, given data on Y, K, H and L, we search for the constant ξ (across all adopting countries and also for different income groups) that minimizes the sum of squared deviations between predicted and observed relative output. Despite the parsimonious parameterization, the fit of the model is remarkably good, indicating that the underlying theory of technological change and diffusion, which places skill endowment, domestic market size and international spillovers as the cornerstone, is broadly consistent with the data. Similarly to Caselli and Coleman (2006), we find that virtually all adopting countries are inside the world technology frontier, that skill scarce countries tend to adopt predominantly low skill-complement innovations and that barriers to adoption are higher in less developed countries. We also find evidence that barriers to technology adoption are relatively stable over the period 1970-2000 among non-OECD economies, while they appear to have fallen for OECD countries. The extreme versions of the model, in which each country develops local technologies independently or in which all country share the same technology, are instead rejected by the data. We also compare the fit of the model under alternative specifications for the cost of adopting technologies that allow us to vary the strength of market size effects and under the assumption of free trade in goods.

With our preferred parameterization, we use the model to perform a series of counterfactuals. First, we show that removing barriers to technology adoption would increase gross domestic product per worker (GDP pw) relative to the US from 0.19 to 0.61 for the average non-OECD country and from 0.68 to 0.91 for the average OECD country. The effect is particularly strong for small countries, which lack the local market size required to benefit from expensive technologies. Second, we study the effect of institutional changes associated to the process of globalization, focusing on the integration of markets for goods and technology. As noted by AZ01 and Acemoglu (2003), trade liberalization may have triggered skill-biased technical change (SBTC) in the US during the last two decades of the 20th century and this may have amplified cross-country income differences. To illustrate the global impact of this phenomenon, we compute the effect both on the world technology frontier and on adopting countries of removing barriers to trade in goods. As trade with skill-scarce countries increase the relative price of skill-intensive goods in the skill-abundant North, it fosters the incentives to introduce skill-complement technologies. The effect on technology adoption is however ambiguous. On the one hand, the increase in the skill bias of the frontier technology makes the adoption of skill-complement technologies cheaper. On the other hand, the rise in the relative price of low-skill-intensive goods in skill-scarce countries promotes the adoption of less skill-biased technologies. We find that, given the estimated parameters, trade would induce most followers to adopt more skill-biased technologies than in the absence of trade. Thus, trade tends to exacerbate the inappropriateness of technologies to the local endowments of non-frontier economies. The result is a global increase in skill premia (a factor of 2.9 for the average country), but also in the cross-country income gap (on average, GDP pw relative to the US falls by 13 percentage points).[3] On the contrary, allowing trade in technology too (i.e., the leader can licence its technology to follower countries), by fostering the incentives to introduce unskilled-labor complement innovations, reduces wage inequality and induces income convergence worldwide.

The paper contributes to a large literature, surveyed in Caselli (2005), aimed at decomposing cross-country income disparities into input differences and unmeasured productivity. We depart from earlier works (e.g., Hall and Jones, 1999) by assuming, consistently with all available evidence, a less than infinite elasticity of substitution between workers of different skill level and by endogenizing productivity. Among more recent contributions, the closest paper is Caselli and Coleman (2006), who also decompose income using the aggregate production function (1.1). There are two main differences, however. First, they back out the pair (A_L, A_H) using data on input, but also factor prices. On the contrary, our theoretical model delivers structural equations that can be used to estimate (1.1) without relying on cross-country factor prices, which are notoriously difficult to obtain for a large sample and not always of high quality. Second, when modelling technology choices, they do not endogenize the world technology frontier. Fadinger (2009) estimates productivity differences across trading countries by fitting both national statistics and the factor content of trade. Yet, he does not endogenize technologies and their diffusion, while in this paper we do not use information contained in trade data.

The paper is also related to the vast literature on international technology diffusion. The idea that countries may benefit from technologies developed elsewhere was first put forward by Nelson and Phelps (1966) and then formalized by Barro and Sala-i-Martin (1997), Howitt (2000) and Acemoglu, Aghion and Zilibotti (2006). Empirical evidence of international technology spillovers is provided, among others, by Keller (2004) and Caselli

[3] We should stress that these very large effects correspond to the extreme experiment of moving from no trade to completely free trade. Clearly, partial trade liberalization would give smaller effects.

and Wilson (2004). Here, we follow closely the model in Gancia and Zilibotti (2009), to which we add capital accumulation. More importantly, one of the main contributions is to estimate the resulting model. The importance of barriers to technology adoption in explaining cross-country income disparities has been emphasized by Parente and Prescott (1994, 2005). Comin and Hobijn (2010) and Comin, Easterly and Gong (2009) document that major innovations diffuse slowly (on average, they are adopted 47 years after their invention), and that differences in the speed of technology adoption are not only large, but also surprisingly persistent over time.

The fact that technologies originating from advanced countries may be excessively skill biased for the endowments of less developed countries, and that this may act both as a barrier to adoption and as source of low productivity, has been put forward by Atkinson and Stiglitz (1969), Basu and Weil (1998), and AZ01. Our approach is most related to AZ01. The main difference is that they only focus on the case in which all countries share the same technology. In the current model, instead, aggregate productivity in less developed countries is relatively low both because of the technology-skill mismatch identified in AZ01 and because of costly adoption.

The paper is structured as follows. Section 2 builds the benchmark model of a world economy where a technology leader engages in directed innovation, while a large number of less advanced countries engage in directed technology adoption. It provides a microfoundation for the aggregate production function (1.1) and illustrates three main sources of low aggregate productivity: lack of capital, distance to frontier and technology inappropriateness. Section 3 extends the model by first allowing trade in goods and then in technology (IPR protection) too. Section 4 estimates the model and quantifies the relative importance of the three sources of income differences. The empirical model is then used to perform counterfactual exercises and sensitivity analysis. Section 5 concludes.

1.2 The Benchmark Model

In this section, we present a model of directed technical change closely related to Acemoglu, Gancia and Zilibotti (2011) and Gancia and Zilibotti (2009). The key ingredients are different types of labor (skilled and unskilled workers), cross-country differences in factor endowments and factor-biased (directed) technical progress. In addition, we consider physical capital accumulation, which was ignored in previous work. Moreover, we emphasize the distinction between the introduction of frontier technologies (*innovation*) which is carried out in the "North", and the sluggish process of imitation and adaptation of such technologies to less developed countries (the "South"). We refer to the latter as technology *adoption.* Adoption is assumed to be cheaper than innovation, creating a laggard advantage. However, since technical change is directed to the factor endowment of the North, the South faces a menu of technologies to imitate that are overly skill biased, given its lower skilled endowment.

1.2.1 Preferences

The world consists of a technology leader (the North), and a set of non-technological leaders (the South), all populated by infinitely lived agents endowed with logarithmic preferences. We denote by N the frontier economy and by $S \in \hat{S} = \{S_1, S_2, ..., S_n\}$ a generic Southern economy. More formally, the utility function of the representative agent in each country is given by:

$$U_J = \int_0^{\infty} e^{-\rho t} \log(c_{Jt}) dt,$$

where $J \in \{N, S\}$ and ρ is the discount rate. The optimal consumption plan satisfies the Euler equation, $\dot{c}_{Jt}/c_{Jt} = r_{Jt} - \rho$, where the interest rate r_{Jt} may be different across countries, since capital markets are not integrated. We remove time indexes when this is no source of confusion.

1.2.2 Technology

Final output, used for both consumption and investment, is produced by a representative competitive firm subject to the following production function:

$$Y_J = K_J^{\alpha} \left[Y_{LJ}^{\frac{\epsilon-1}{\epsilon}} + Y_{HJ}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon(1-\alpha)}{\epsilon-1}}, \tag{1.2}$$

where K is capital, Y_L and Y_H are intermediate goods produced with unskilled and skilled labor, respectively, and $\epsilon > 1$ is the elasticity of substitution between them. Profit maximization implies that the rental rate of capital equals the marginal product of capital. More formally, after choosing Y as the *numéraire*, we have $K_J = \alpha Y_J / (r_J \chi_J)$, where χ_J is a wedge capturing distortions which open a gap between the private and social rate of returns to investments. When $\chi_J = 1$, there is no distortion, and the standard condition equating the interest rate to the marginal product of capital holds. Substituting back K_J into (1.2) yields:

$$Y_J = \left(\frac{\alpha}{r_J \chi_J} \right)^{\frac{\alpha}{1-\alpha}} \left[Y_{LJ}^{\frac{\epsilon-1}{\epsilon}} + Y_{HJ}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}. \tag{1.3}$$

Profit maximization implies then the following inverse demand functions:

$$P_{LJ} = (1-\alpha)\left(\frac{\alpha}{r_J \chi_J}\right)^{\frac{(\epsilon-1)\alpha}{\epsilon(1-\alpha)}}\left(\frac{Y_J}{Y_{LJ}}\right)^{\frac{1}{\epsilon}} \text{ and}$$
$$P_{HJ} = (1-\alpha)\left(\frac{\alpha}{r_J \chi_J}\right)^{\frac{(\epsilon-1)\alpha}{\epsilon(1-\alpha)}}\left(\frac{Y_J}{Y_{HJ}}\right)^{\frac{1}{\epsilon}}, \tag{1.4}$$

where P_L and P_H are the prices of Y_L and Y_H, respectively. Note that $P_{HJ}/P_{LJ} = [Y_{LJ}/Y_{HJ}]^{\frac{1}{\epsilon}}$.

The production function at the sector level is given by:

$$Y_{LJ} = E_{LJ}\left[\int_0^{A_{LJ}} y_{LJ}(i)^{\frac{\sigma-1}{\sigma}}\, di\right]^{\frac{\sigma}{\sigma-1}} \quad \text{and} \quad Y_{HJ} = E_{HJ}\left[\int_0^{A_{HJ}} y_{HJ}(i)^{\frac{\sigma-1}{\sigma}}\, di\right]^{\frac{\sigma}{\sigma-1}},$$

where (A_L, A_H) is the state vector consisting of the measure of intermediate inputs produced with unskilled and skilled labor, respectively. The terms $E_{LJ} \equiv (A_{LJ})^{\frac{\sigma-2}{\sigma-1}}$ and $E_{HJ} \equiv (A_{HJ})^{\frac{\sigma-2}{\sigma-1}}$ are externalities that make the model consistent with the existence of a balanced growth path (see Gancia and Zilibotti (2009) for a discussion of such externalities). Note that the externality vanishes at $\sigma = 2$.

The producers of Y_L and Y_H are also competitive. Their profit maximization yields the following relative demand equations:

$$\frac{y_{LJ}(i)}{y_{LJ}(j)} = \left[\frac{p_{LJ}(j)}{p_{LJ}(i)}\right]^{\sigma} \quad \text{and} \quad \frac{y_{HJ}(i)}{y_{HJ}(j)} = \left[\frac{p_{HJ}(j)}{p_{HJ}(i)}\right]^{\sigma},$$

where p_L and p_H denote the price of intermediates.

The intermediate good sector is monopolistic, with each producer holding the patent for a single variety. The production function for each intermediate input, $y_{LJ}(i)$ and $y_{HJ}(i)$, is linear in the type of labor employed,

$$y_{LJ}(i) = l_J(i) \text{ and } y_{HJ}(i) = Z h_J(i),$$

where $Z \geq 1$ is a parameter that will allow us to match the level of the skill premium in the benchmark case. The industry equilibrium is subject to the resource constraints $\int_0^{A_{LJ}} l_J(i)\, di \leq L_J$ and $\int_0^{A_{HJ}} h_J(i)\, di \leq H_J$, where L_J and H_J are assumed to be in fixed supply. As the monopolists face a demand curve with the constant price elasticity of σ, it is optimal for them to set prices equal to $p_{LJ}(i) = p_{LJ} = (1-1/\sigma)^{-1} w_{LJ}$ and $p_{HJ}(i) = p_{HJ} = (1-1/\sigma)^{-1} w_{HJ}/Z$, where w_L and w_H are the wage of unskilled and skilled workers, respectively. This pricing formula also implies that profits per firm are a fraction $1/\sigma$ of revenues:

$$\pi_{LJ}(i) = \frac{p_{LJ} l_J(i)}{\sigma} \quad \text{and} \quad \pi_{HJ}(i) = \frac{p_{HJ} Z h_J(i)}{\sigma}. \tag{1.5}$$

Using symmetry and labor market clearing yields $l_J(i) = L_J/A_{LJ}$ and $h_J(i) = H_J/A_{HJ}$, which in turn allows to express sectorial output as:

$$Y_{LJ} = A_{LJ} L_J \quad \text{and} \quad Y_{HJ} = A_{HJ} Z H_J. \tag{1.6}$$

Note that output in each sector is a linear function of labor and of the state of technology. Plugging (1.6) into (1.4) yields the relative price:

$$\tilde{P}_J \equiv \frac{P_{HJ}}{P_{LJ}} = \left[\tilde{A}_J Z \tilde{h}_J\right]^{-\frac{1}{\epsilon}}, \tag{1.7}$$

where $\tilde{A} \equiv A_H/A_L$ is the skill bias of the technology and $\tilde{h} \equiv H/L$ is the relative skill endowment. Note that "tilde" denotes relative (skill-to-unskill) variables. Relative wages and profits can be found using (1.7), and noting that $p_{LJ} L_J = P_{LJ} Y_{LJ}$ and $p_{HJ} Z H_J = P_{HJ} Y_{HJ}$:

$$\tilde{w}_J \equiv \frac{w_{HJ}}{w_{LJ}} = Z \frac{P_{HJ}}{P_{LJ}} \frac{A_{HJ}}{A_{LJ}} = \left[Z \tilde{A}_J\right]^{1-\frac{1}{\epsilon}} \left[\tilde{h}_J\right]^{-\frac{1}{\epsilon}} \tag{1.8}$$

$$\tilde{\pi}_J \equiv \frac{\pi_{HJ}}{\pi_{LJ}} = \frac{P_{HJ}}{P_{LJ}} \frac{Z H_J}{L_J} = \tilde{A}_J^{-\frac{1}{\epsilon}} \left(Z \tilde{h}_J\right)^{1-\frac{1}{\epsilon}}. \tag{1.9}$$

Equation (1.9) shows that the relative profitability, π_H/π_L, has two components: a "price effect", whereby rents are higher in sectors producing more expensive goods, and a "market size effect", whereby rents are higher in bigger sectors.

1.2.3 Innovation in the North

Frontier innovation is carried out in the North, and takes the form of the introduction of new varieties of intermediate inputs. We assume that the development of *any* new variety requires a fixed cost of μ units of the numéraire Y. The direction of innovation is endogenous, i.e., each innovator can decide to design a variety that can be used in the H or L sector. As patents are infinitely lived, the value of a firm – either V_L or V_H – is the present discounted value of its future profit stream. Free entry implies that neither V_L nor V_H can exceed the innovation cost, μ. Since in a balanced growth path (a *steady state*) P_L, P_H and the interest rate r are constant, then $V_{LN} = \pi_{LN}/r_N = V_{HN} = \pi_{HN}/r_N = \mu$, which implies in turn that $\tilde{\pi}_N = 1$. The equalization of profit flows yields the equilibrium skill bias of technology in the North:

$$\tilde{A}_N = \left(Z\tilde{h}_N \right)^{\epsilon-1}. \tag{1.10}$$

Substituting $\tilde{A}_N$ into (1.8) yields the steady-state skill premium:

$$\tilde{w}_N = Z^{\epsilon-1} \left(\tilde{h}_N \right)^{\epsilon-2}. \tag{1.11}$$

To find the growth rate, we note that the interest rate is pinned down by either of the two free entry conditions, e.g., $r_N = \pi_{HN}/\mu = P_{HN}ZH_N/(\mu\sigma)$. Using (1.3), (1.4) and (1.6) to eliminate P_{HN}, normalizing $\chi_N = 1$, and using the Euler equation yields the balanced growth rate of the economy,

$$g_N = r_N - \rho = (1-\alpha)^{1-\alpha} \alpha^{\alpha} \left[\frac{L_N^{\epsilon-1} + (ZH_N)^{\epsilon-1}}{(\sigma\mu)^{\epsilon-1}} \right]^{\frac{1-\alpha}{\epsilon-1}} - \rho. \tag{1.12}$$

It can be shown that, along the balanced growth path, Y_N, c_N, K_N, A_{HN} and A_{LN} all grow at the rate g_N.

1.2.4 Directed Technology Adoption in the South

Southern countries are assumed to be skill scarce, namely, $\tilde{h}_S < \tilde{h}_N$ for all $S \in \hat{S}$, and to start from a lower technology level in both the skilled and unskilled sector. They can adopt at a cost the technologies developed in the North. To begin with, we assume that there is neither trade in goods nor international protection of IPR. Each of these assumption will be relaxed later on. The lack of IPR implies that innovators in the North cannot sell their copyrights to firms located in the South, so that the only market they have access to is the domestic one. In the absence of trade, the equilibrium conditions in the North are unaffected by the presence of the South.

The equilibrium conditions of Southern economies are analogous to those of the North, except for technology adoption, which differs from the innovation process. In particular, Southern countries take the state of the frontier technology, A_{LN} and A_{HN}, as given. Technology adoption is modeled as a costly investment activity similar to innovation. Following the earlier literature, we assume that, due to technological spillovers, the cost of adopting a technology in a sector, c_{LS} and c_{HS}, is a negative function of the technological gap in that sector:

$$c_{LS} = \mu \left(\frac{A_{LS}}{A_{LN}} \right)^{\xi} \quad \text{and} \quad c_{HS} = \mu \left(\frac{A_{HS}}{A_{HN}} \right)^{\xi}, \quad \xi \geq 0, \tag{1.13}$$

where A_{LN} and A_{HN} represent the world technology frontiers in the two sectors. That is, the farther behind a country is relative to the skill-specific frontier, the cheaper it is to adopt technologies in that sector. With this formulation, the total cost of adopting the entire set of z-complement technologies (with $z \in \{H, L\}$) is:

$$\mu \int_0^{A_{zN}} \left(\frac{A_{zS}}{A_{zN}} \right)^{\xi} dA_{zS} = \frac{\mu A_{zN}}{1 + \xi}.$$

This expression shows that ξ can be interpreted as an inverse measure of barriers to technology adoption in the South. All intermediate inputs adopted in the South are sold by local monopolists.

In steady state, free entry implies $\pi_{HS}/\pi_{LS} = c_{HS}/c_{LS}$. Using this condition together with equations (1.9), (1.10) and (1.13), we can solve for the skill bias of the technology in the South:

$$\tilde{A}_S = \left(Z\tilde{h}_S \right)^{\frac{\epsilon-1}{1+\epsilon\xi}} \tilde{A}_N^{\frac{\epsilon\xi}{1+\epsilon\xi}} = Z^{\epsilon-1} \left[\tilde{h}_S \cdot \tilde{h}_N^{\epsilon\xi} \right]^{\frac{\epsilon-1}{1+\epsilon\xi}}. \tag{1.14}$$

Technology adoption in the South depends on the skill endowment of the North and of the local economy. On the one hand, local skill abundance increases the profitability of adopting skill-complement innovations. On the other hand, skill abundance in the North means that the frontier technology is more skill biased, and that skilled technologies are cheaper to imitate. Note also that the skill bias of the technology in the adopting economy is increasing in ξ, capturing the speed of technology transfer. In particular, in the limit case of $\xi = 0$ (prohibitive barriers) each economy develops local technologies independently from the world frontier, and the skill abundance in the North becomes irrelevant: $\tilde{A}_S = \left(Z\tilde{h}_S \right)^{\epsilon-1}$. To the opposite case, as $\xi \to \infty$, adoption is free so that the South is using the technology of the North. In this case, it is the local skill endowment that does not matter: $\tilde{A}_S = \tilde{A}_N = \left(Z\tilde{h}_N \right)^{\epsilon-1}$. The latter is the case analyzed by AZ01.

1.2.5 Productivity Differences

As long as $\xi > 0$, a balanced growth path features $r_S = r_N \equiv r$, with the South and the North growing at the same rate, in spite of there being neither trade nor factor mobility. The model yields then predictions for steady-state output and productivity differences as functions of factor endowments and of exogenous parameters.

Proposition 1 *For any $S \in \hat{S}$, the steady-state output ratio relative to the frontier is*

$$\frac{Y_S}{Y_N} = \left(\left(\frac{K_S}{K_N}\right)^{\alpha} \left[\frac{L_S^{\frac{(\epsilon-1)(1+\xi)}{1+\epsilon\xi}} + \left(Z\tilde{h}_N\right)^{\frac{\xi(\epsilon-1)^2}{1+\epsilon\xi}} \times (ZH_S)^{\frac{(\epsilon-1)(1+\xi)}{1+\epsilon\xi}}}{L_N^{\frac{(\epsilon-1)(1+\xi)}{1+\epsilon\xi}} + \left(Z\tilde{h}_N\right)^{\frac{\xi(\epsilon-1)^2}{1+\epsilon\xi}} \times (ZH_N)^{\frac{(\epsilon-1)(1+\xi)}{1+\epsilon\xi}}} \right]^{\frac{(1-\alpha)(1+\epsilon\xi)}{(\epsilon-1)(1+\xi)}} \right)^{\frac{1+\xi}{\alpha+\xi}}$$
$$\equiv f_S^{AUT}, \tag{1.15}$$

where $K_S/K_N = (Y_S/Y_N)/(\chi_S/\chi_N)$.

Proof: The production function, (1.2), yields

$$\frac{Y_S}{Y_N} = \left(\frac{A_{LS}}{A_{LN}}\right)^{1-\alpha} \left(\frac{K_S}{K_N}\right)^{\alpha} \left[\frac{L_S^{\frac{\epsilon-1}{\epsilon}} + \tilde{A}_S^{\frac{\epsilon-1}{\epsilon}} (ZH_S)^{\frac{\epsilon-1}{\epsilon}}}{L_N^{\frac{\epsilon-1}{\epsilon}} + \tilde{A}_N^{\frac{\epsilon-1}{\epsilon}} (ZH_N)^{\frac{\epsilon-1}{\epsilon}}} \right]^{\frac{\epsilon(1-\alpha)}{\epsilon-1}}. \tag{1.16}$$

To obtain the equilibrium expression for A_{LS}/A_{LN}, recall first that

$$\frac{\pi_{LS}}{\pi_{LN}} = \frac{c_{LS}}{\mu} = \left(\frac{A_{LS}}{A_{LN}}\right)^{\xi} \text{ and } \frac{\pi_{HS}}{\pi_{HN}} = \frac{c_{HS}}{\mu} = \left(\frac{A_{HS}}{A_{HN}}\right)^{\xi}, \tag{1.17}$$

where the relative profits can be written as

$$\frac{\pi_{LS}}{\pi_{LN}} = \frac{P_{LS} Y_{LS} A_{LN}}{P_{LN} Y_{LN} A_{LS}} = \frac{P_{LS} L_S}{P_{LN} L_N}, \tag{1.18}$$

using (1.5) and (1.6). Next, note that, since the price of Y_L equals its marginal product, then:

$$\frac{P_{LS}}{P_{LN}} = \left(\frac{A_{LS}}{A_{LN}}\right)^{-\alpha} \left(\frac{K_S}{K_N}\right)^{\alpha} \left[\frac{L_S^{\frac{\epsilon-1}{\epsilon}} + \tilde{A}_S^{\frac{\epsilon-1}{\epsilon}} (ZH_S)^{\frac{\epsilon-1}{\epsilon}}}{L_N^{\frac{\epsilon-1}{\epsilon}} + \tilde{A}_N^{\frac{\epsilon-1}{\epsilon}} (ZH_N)^{\frac{\epsilon-1}{\epsilon}}} \right]^{\frac{\epsilon(1-\alpha)}{\epsilon-1}-1} \left(\frac{L_S}{L_N}\right)^{-\frac{1}{\epsilon}}. \tag{1.19}$$

Next, (1.17), (1.18) and (1.19) imply that:

$$\frac{A_{LS}}{A_{LN}} = \left(\frac{L_S}{L_N}\right)^{\frac{\epsilon-1}{\epsilon(\xi+\alpha)}} \left(\frac{K_S}{K_N}\right)^{\frac{\alpha}{\xi+\alpha}} \left[\frac{L_S^{\frac{\epsilon-1}{\epsilon}} + \tilde{A}_S^{\frac{\epsilon-1}{\epsilon}} (ZH_S)^{\frac{\epsilon-1}{\epsilon}}}{L_N^{\frac{\epsilon-1}{\epsilon}} + \tilde{A}_N^{\frac{\epsilon-1}{\epsilon}} (ZH_N)^{\frac{\epsilon-1}{\epsilon}}} \right]^{-\frac{\alpha\epsilon-1}{(\epsilon-1)(\xi+\alpha)}}. \tag{1.20}$$

We can now use (1.20) to substitute away A_{LS}/A_{LN} into (1.16):

$$\frac{Y_S}{Y_N} = \left(\frac{L_S}{L_N}\right)^{\frac{(1-\alpha)(\epsilon-1)}{\epsilon(\xi+\alpha)}} \left(\frac{K_S}{K_N}\right)^{\alpha\frac{1+\xi}{\alpha+\xi}} \left[\frac{L_S^{\frac{\epsilon-1}{\epsilon}} + \tilde{A}_S^{\frac{\epsilon-1}{\epsilon}} (ZH_S)^{\frac{\epsilon-1}{\epsilon}}}{L_N^{\frac{\epsilon-1}{\epsilon}} + \tilde{A}_N^{\frac{\epsilon-1}{\epsilon}} (ZH_N)^{\frac{\epsilon-1}{\epsilon}}} \right]^{\frac{(1-\alpha)(1+\epsilon\xi)}{(\epsilon-1)(\alpha+\xi)}}. \tag{1.21}$$

Finally, eliminating $\tilde{A}_N$ and $\tilde{A}_S$ from (1.21) using (1.10) and (1.14), respectively, and rearranging terms, yields (1.15). ■

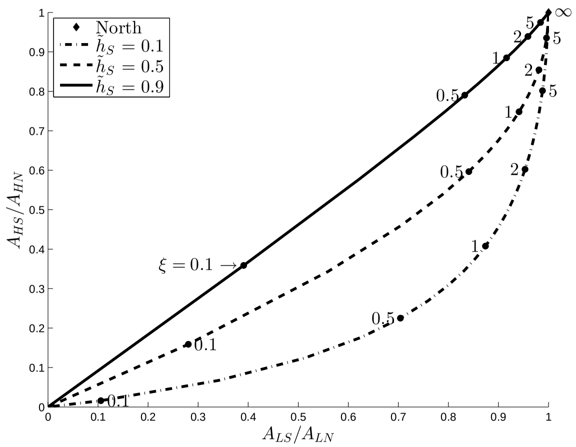
The formula of the output gap (1.15) resembles the ratio between two identical aggregate constant elasticity of substitution (CES) production functions. This is remarkable, since countries use in fact different technologies. However, the implied production function differs from standard CES functions such as (1.1) in two respects: First, it features increasing returns to scale, parameterized by the exponent $(1+\xi)/(\alpha+\xi) > 1$. Second, the structural parameter ξ implies a particular restriction between the *long-run* elasticity of substitution between high- and low-skill labor and the "weight" of the CES function, $\left(Z\tilde{h}_N\right)^{\frac{\xi(\epsilon-1)^2}{1+\epsilon\xi}}$. Given the structural parameters ϵ, α, ξ and Z, the right-hand side of the relative GDP equation is fully determined by the data of capital, low-skill labor and high-skill labor. Dividing both sides by the number of agents (workers) yields an accounting equation for GDP per capita (per worker).

As noted above, income differences depend on a scale effect, namely, larger countries are predicted to be *ceteris paribus* more productive. Interestingly, this effect disappears as barriers to adoption vanish and all countries converge to the technology frontier. Indeed, as $\xi \to \infty$, we have that $\tilde{A}_S \to \tilde{A}_N$ and

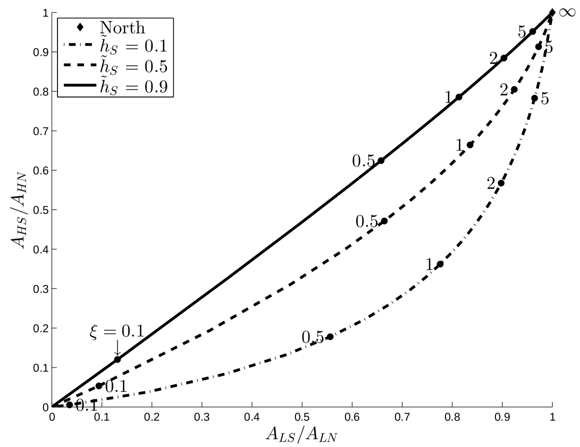
$$\lim_{\xi\to\infty} \frac{Y_S}{Y_N} = \left(\frac{K_S}{K_N}\right)^{\alpha} \left[\frac{L_S^{\frac{\epsilon-1}{\epsilon}} + \left(Z\tilde{h}_N\right)^{\frac{(\epsilon-1)^2}{\epsilon}} \times (ZH_S)^{\frac{\epsilon-1}{\epsilon}}}{L_N^{\frac{\epsilon-1}{\epsilon}} + \left(Z\tilde{h}_N\right)^{\frac{(\epsilon-1)^2}{\epsilon}} \times (ZH_N)^{\frac{\epsilon-1}{\epsilon}}}\right]^{\frac{\epsilon(1-\alpha)}{\epsilon-1}}, \tag{1.22}$$

which is the equation estimated by AZ01, who also set $\epsilon = 2$.

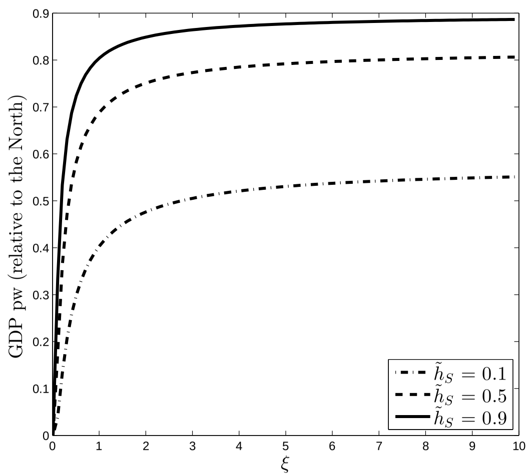
Figure 1.1 shows how different parameters affect productivity differences. The figure depicts economies with equally sized total labor forces and with $\epsilon = 2$. The parameters of the North are fixed at $\tilde{h}_N = \chi_N = 1$, and $Z = 1.5$, implying that $A_{HN}/A_{LN} = 1.5$. Then, we consider Southern economies with different skill endowments, $\tilde{h}_S$, barriers to technology adoption, ξ, and investment wedges, χ_S. Panel (a) shows the pattern of technology adoption, i.e., the equilibrium proximity to the frontier in the L and H sector, respectively for different combinations of ξ and $\tilde{h}_S$, while holding constant $\chi_S = 1.2$. The figure plots three curves, each corresponding to a different relative skill endowment: $\tilde{h}_S = 0.9$ (highest curve), $\tilde{h}_S = 0.5$ (intermediate curve) and $\tilde{h}_S = 0.1$ (lowest curve). Moving along each curve from left to right yields points with increasing ξ. Dots single out some particular values of ξ. The parameter ξ affects both the distance to frontier (lower ξ implies a larger gap) and the skill bias of technology adoption. In particular, the lower ξ the more the technology will reflect local conditions. As we increase ξ the technology becomes more skill biased, as one



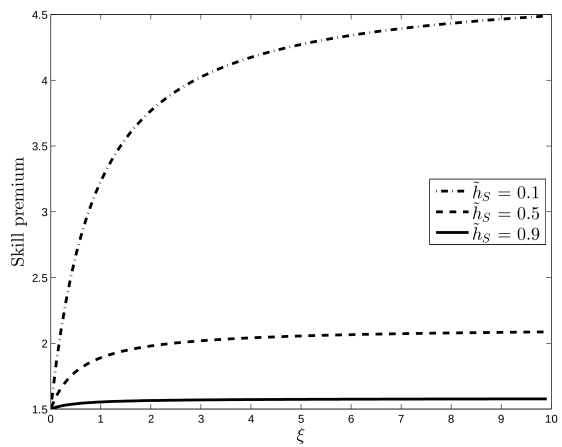
(a) $\chi_S/\chi_N = 1.2$



(b) $\chi_S/\chi_N = 1.5$



(c) $\chi_S/\chi_N = 1.2$



(d) $\chi_S/\chi_N = 1.2$

Figure 1.1: Steady-state comparative statics.

can see by drawing rays from the origin through different dotted points along a line. For large levels of ξ, the technological differences between non-frontier economies with different endowments become very small, and are all well approximated by the case studied by AZ01 in which all countries adopt immediately the frontier technology. Panel (b) shows the same combination of parameters, but with a larger investment wedge $\chi_S = 1.5$. A larger χ_S reduces technology adoption, especially for countries with higher skill ratios. For example, a country with $\tilde{h}_S = 0.5$ and $\xi = 2$ adopts 85% of the high-skill and 98% of the low-skill technologies if $\chi_S = 1.2$, while it adopts 80% of the high-skill and 92% of the low-skill technologies if $\chi_S = 1.5$.

Panels (c) and (d) display the effect of ξ and $\tilde{h}_S$ on output per worker differences and the skill premium. As in panel (a), the investment wedge is fixed at $\chi_S = 1.2$ and each of the three curves represents a different $\tilde{h}_S$. Panel (c) shows that, as long as $\xi < 2$, barriers to technology adoption are important. However, for larger values of ξ the lion share of productivity differences originates from technology inappropriateness, i.e., the excessive skill bias of frontier technologies. For instance, if $\tilde{h}_S = 0.1$ and $\xi = 2$, removing all barriers would only reduce 18 percent of the distance to the frontier. In contrast, 74 percent of the productivity gap is due to technology mismatch, and 14 percent is due to the investment wedge. Moving back to panel (a), one can note that in this case more than 90 percent of the technologies used by low-skill workers are already in use when $\xi = 2$ and $\tilde{h}_S = 0.1$. Thus, slashing barriers triggers mainly the adoption of high-skill technologies (when $\xi = 2$ the Southern economy only adopts 60 percent of the high-skill technologies). However, this yields only modest productivity gains since only about 11% of the labor force is skilled.

The skill bias of technology is reflected in the wage inequality. The steady-state skill premium is given by $\tilde{w}_S = Z^{\epsilon-1}\tilde{h}_S^{\frac{\epsilon-\xi-2}{1+\epsilon\xi}}\tilde{h}_N^{\frac{(\epsilon-1)^2\xi}{1+\epsilon\xi}}$, where $\tilde{w}_S$ is increasing in ξ, ranging from $\tilde{w}_{S|\xi=0} = Z^{(\epsilon-1)}\tilde{h}_S^{\epsilon-2}$ to $\tilde{w}_{S|\xi\to\infty} = Z^{(\epsilon-1)}\tilde{h}_S^{-\frac{1}{\epsilon}}h_N^{\frac{(\epsilon-1)^2}{\epsilon}}$. Panel (d) of Figure 1 shows the long-run effect of ξ on wages for alternative relative skill endowments in the South. Increasing ξ induces a rise in the skill premium which is a direct consequence of the previous finding that a higher relative fraction of high-skill technologies are adopted as ξ increases. Moreover, starting from $\xi = 2$, the rise in the skill premium is steeper in countries with low skill ratios because there are more high-skill technologies left to adopt.

1.3 Extensions: Trade and IPR

So far, we have only allowed countries to interact through technological spillovers. In this section we extend the analysis first to economies that trade in goods and then to economies that, in addition, can import technologies through licensing contracts. We refer to the latter case as full IPR enforcement.

1.3.1 International Trade

In this section, we assume that the intermediate good Y_L and Y_H can be traded internationally without frictions. Under free trade, there is a single world price for P_L and P_H,

$$\tilde{P}^w \equiv \frac{P_H^w}{P_L^w} = \left[\frac{Y_L^w}{Y_H^w}\right]^{\frac{1}{\epsilon}}, \tag{1.23}$$

where the superscript w refer to worldwide variables. Hence, $Y_L^w = A_{LN}L_N + \sum_{j=1}^n A_{LS_j}L_{S_j}$ and $Y_H^w = A_{HN}ZH_N + \sum_{j=1}^n A_{HS_j}ZH_{S_j}$. All equations in Section 1.2.2 continue to hold, with local prices being now equal to the world price.

Consider, next, the innovation process in the North. The key observation is that the North continues to be the relevant market for new frontier technologies, since there is no IPR protection in the South. The profit flows of Northern firms are, then, $\pi_{LN} = P_L^w L_N/\sigma$ and $\pi_{HN} = P_H^w ZH_N/\sigma$. In a balanced-growth equilibrium, $\tilde{\pi}_N = 1$, which in turn implies that $\tilde{P}^w = \left(Z\tilde{h}_N\right)^{-1}$. Using (1.23) and rearranging terms (see proof below) leads to the following Lemma.

Lemma 1 *In a free trade environment, the skill bias of the frontier technology is given by:*

$$\tilde{A}_N = \tilde{A}_N^{trade} \equiv \frac{\left(Z\tilde{h}_N\right)^{\epsilon-1}}{\hat{h}} > \left(Z\tilde{h}_N\right)^{\epsilon-1}, \tag{1.24}$$

where

$$\hat{h} \equiv \frac{1 + \sum_{j=1}^n \left(\frac{H_{S_j}}{H_N}\right)^{\frac{1+\xi}{\xi}}}{1 + \sum_{j=1}^n \left(\frac{L_{S_j}}{L_N}\right)^{\frac{1+\xi}{\xi}}} < 1. \tag{1.25}$$

The skill bias of technology in country $S \in \hat{S}$ is given by

$$\tilde{A}_S = \tilde{A}_S^{trade} \equiv \frac{\left(Z\tilde{h}_N\right)^{\epsilon-1}}{\hat{h}} \left(\frac{\tilde{h}_S}{\tilde{h}_N}\right)^{\frac{1}{\xi}}. \tag{1.26}$$

Proof: Using (1.23) to substitute away $\tilde{P}^w$ from the equation $\tilde{P}^w = \left(Z\tilde{h}_N\right)^{-1}$ yields:

$$Z\tilde{h}_N = \left[Z\tilde{A}_N \left(\frac{\sum_{j=1}^n \frac{A_{HS_j}}{A_{HN}} H_{S_j} + H_N}{\sum_{j=1}^n \frac{A_{LS_j}}{A_{LN}} L_{S_j} + L_N} \right) \right]^{\frac{1}{\epsilon}}.$$

Solving out for $\tilde{A}_N$ yields:

$$\tilde{A}_N = \left(Z\tilde{h}_N\right)^{\epsilon-1} \times \left(\frac{1 + \sum_{j=1}^n \frac{A_{LS_j}}{A_{LN}} \frac{L_{S_j}}{L_N}}{1 + \sum_{j=1}^n \frac{A_{HS_j}}{A_{HN}} \frac{H_{S_j}}{H_N}} \right) \equiv \tilde{A}_N^{trade}. \tag{1.27}$$

We must now solve for the skill-specific distance-to-frontier terms. To this aim, note that, on the one hand, $\pi_{HS}/\pi_{HN} = H_S/H_N$ and $\pi_{LS}/\pi_{LN} = L_S/L_N$. On the other hand, in a balanced growth path, $\pi_{HS}/\pi_{HN} = c_{HS}/\mu$ and $\pi_{LS}/\pi_{LN} = c_{LS}/\mu$. Thus, $c_{HS} = \mu H_S/H_N$ and $c_{LS} = \mu L_S/L_N$. Then, using (1.13) to eliminate c_{HS} and c_{LS} yields:

$$\frac{A_{HS}}{A_{HN}} = \left(\frac{H_S}{H_N}\right)^{1/\xi} \quad \text{and} \quad \frac{A_{LS}}{A_{LN}} = \left(\frac{L_S}{L_N}\right)^{1/\xi}. \tag{1.28}$$

Plugging (1.28) into (1.27) yields (1.24). Finally, (1.26) follows immediately from (1.24) and (1.28). ■

The numerator of (1.24) is identical to its no-trade counterpart, (1.10). The denominator is smaller than unity, since Southern economies are skill scarce relative to the North. Thus, trade increases the skill bias of the frontier technology. This result generalizes the finding of AZ01 to an environment in which technology adoption is costly. Equation (1.24) also shows that the "trade multiplier" depends on ξ and on the relative market size and skill endowment of the two economies. $\tilde{A}_N$ increases with the difference in the skill endowment between the North and the South. Trade increases the relative price of the good that is intensive in the factor that is relatively abundant in each country (i.e., $\tilde{P}$ in the North) and the effect is larger the more different factor endowments are. Then, the stronger the increase in $\tilde{P}$ in the North relative to the no-trade environment, the larger the skill bias induced by trade. Barriers (i.e., a reduction in ξ) increase $\tilde{A}_N$. The intuition behind this result is that since the frontier technology is skill biased, technology transfer reduces the difference in effective endowments. In other words, barriers reduce the skill bias of adoption, thereby strengthening the North-South pattern of specialization in production. As a consequence, the price effect is larger when barriers are higher.

The effect of trade on the direction of technology adoption in the South (equation (1.26)) is instead ambiguous. On the one hand, trade increases the relative price of low-skill-intensive goods in the South, accelerating the adoption of low-skill technologies. On the other hand, the higher skill bias at the frontier makes it cheaper to adopt skilled technologies.[4]

The following proposition provides an expression for output differences – the analogue of equation (1.15) – under free trade.

Proposition 2 *Assume free international trade in the intermediate goods* Y_H *and* Y_L*. For any* $S \in \hat{S}$*, the steady-state output ratio relative to the frontier is:*

$$\frac{Y_S}{Y_N} = \left(\frac{K_S}{K_N}\right)^{\alpha} \left(\frac{L_S^{\frac{1+\xi}{\xi}} + \frac{\left(Z\tilde{h}_N\right)^{\frac{\xi(\epsilon-1)-(1+\xi)}{\xi}}}{\hat{h}} \times (ZH_S)^{\frac{1+\xi}{\xi}}}{L_N^{\frac{1+\xi}{\xi}} + \frac{\left(Z\tilde{h}_N\right)^{\frac{\xi(\epsilon-1)-(1+\xi)}{\xi}}}{\hat{h}} \times (ZH_N)^{\frac{1+\xi}{\xi}}} \right)^{1-\alpha} \equiv f_S^{trade}, \tag{1.29}$$

where $K_S/K_N = (Y_S/Y_N)/(\chi_S/\chi_N)$*.*

Proof: Rewrite the production function as $Y_J = (K_J)^{\alpha}\left(\hat{Y}_J\right)^{1-\alpha}$, where we define $\hat{Y}_J \equiv \left[\hat{Y}_{LJ}^{\frac{\epsilon-1}{\epsilon}} + \hat{Y}_{HJ}^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}$ and $\hat{Y}_{LJ}$ and $\hat{Y}_{HJ}$ denote the quantities used in final production in country J. Due to trade, these quantities differ from the respective local production levels (which we continue to denote by Y_{LJ} and Y_{HJ}). Balanced trade implies that

$$P_{\hat{Y}}^{w}\hat{Y}_J = P_{HJ}^{w}\hat{Y}_{HJ} + P_{LJ}^{w}\hat{Y}_{LJ} = P_{HJ}^{w}A_{HJ}ZH_J + P_{LJ}^{w}A_{LJ}L_J, \tag{1.30}$$

where $P_{\hat{Y}}^{w} = \left[(P_L^w)^{1-\epsilon} + (P_H^w)^{1-\epsilon}\right]^{1/(1-\epsilon)}$ is the same for all countries. Thus, for any $S \in \hat{S}$, we can write:

$$\frac{Y_S}{Y_N} = \left(\frac{K_S}{K_N}\right)^{\alpha}\left(\frac{\hat{Y}_S}{\hat{Y}_N}\right)^{1-\alpha} = \left(\frac{K_S}{K_N}\right)^{\alpha}\left(\frac{P_H^w A_{HS}ZH_S + P_L^w A_{LS}L_S}{P_H^w A_{HN}ZH_N + P_L^w A_{LN}L_N}\right)^{1-\alpha}, \tag{1.31}$$

where the second equality comes from (1.30) and the fact that $\hat{Y}_S/\hat{Y}_N = P_{\hat{Y}}^{w}\hat{Y}_S/\left(P_{\hat{Y}}^{w}\hat{Y}_N\right)$. Rearranging terms yields:

$$\frac{Y_S}{Y_N} = \left(\frac{K_S}{K_N}\right)^{\alpha}\left(\frac{A_{LS}L_S}{A_{LN}L_N} \cdot \frac{1+\tilde{P}^w\tilde{A}_S Z\tilde{h}_S}{1+\tilde{P}^w\tilde{A}_N Z\tilde{h}_N}\right)^{1-\alpha}. \tag{1.32}$$

Then, using (1.28) and (1.23) to eliminate A_{LS}/A_{LN} and $\tilde{P}^w$, respectively, and rearranging terms, yields (1.29). ■

[4]More formally, $\tilde{A}_S^{trade}/\tilde{A}_S = \hat{h}^{-1}\left(\tilde{h}_S/\tilde{h}_N\right)^{\frac{\xi+1}{\xi(\epsilon\xi+1)}}$, showing that trade increases (decreases) the skill bias of technology adoption if ξ is sufficiently large (small).

As emphasized in Ventura (2005) and Fadinger (2009), trade affects the shape of countries' aggregate production possibility frontier. In particular, for given technology, the elasticity of substitution between Y_{LS} and Y_{HS} $\left(\text{equivalently, between } L_S^{(1+\xi)/\xi} \text{ and } H_S^{(1+\xi)/\xi}\right)$ is now infinite, instead of ϵ, because all countries face the same world prices. The exponent $(1+\xi)/\xi \geq 1$ still captures the extent of the scale effect in adoption.

1.3.2 IPR (Licensing of Technologies)

In this section, we maintain free trade and also allow frontier technologies to be licensed from Northern to Southern (monopolist) firms in exchange of a perpetual royalty per unit produced in the South. For simplicity, we assume that when a technology is licensed there are no additional adoption costs. While some local firms could in principle choose to adopt frontier technologies that have not yet been licensed, in equilibrium all technologies will be licensed to the South as soon as they are introduced in the North.[5] Thus, no room is left for unlicensed technology adoption. Intuitively, this follows from the assumption that innovators can transfer technologies at zero costs. Therefore, no matter how low the cost of unlicensed adoption is, Northern producers will bid down the license cost and win the race. The discussion is summarized by the following Lemma.

Lemma 2 *Suppose that Northern producers can license their technology. Then, there exists a unique subgame perfect Nash equilibrium such that the South adopts instantaneously all technologies introduced in the North. All profits made in the Southern market are transferred to Northern firms as royalties.*

Full IPR protection entails both costs and benefits for the South. On the one hand, the South must transfer to the North the entire profit flow of intermediate producers. On the other hand, the South can adopt immediately all technologies (similar to the case of $\xi \to \infty$ in the benchmark model). In addition, IPR enforcement affects the direction of technical change, reducing the skill bias of the frontier technology. To see this, note that in steady state the present discounted value of the royalties paid by country S_j are $\varphi_{LS_j} = \pi_{LS_j}/r$ and $\varphi_{HS} = \pi_{HS_j}/r$. Including royalties, the zero-profit conditions for innovation yield:

$$\mu - \sum_{j=1}^{n} \varphi_{LS_j} = \frac{\pi_{LN}}{r}, \text{ and } \mu - \sum_{j=1}^{n} \varphi_{HS_j} = \frac{\pi_{HN}}{r}.$$

[5] After a technology has been licensed to a firm in country S, there is no reason for a firm to pay a cost to produce the same variety, since Bertrand competition would bring the profit of the entrant first to zero.

The equilibrium skill bias, $\tilde{A}_N$ (where $\tilde{A}_S = \tilde{A}_N$), is the determined implicitly by the following equation:

$$1 = \frac{\pi_{HN} + \sum_{j=1}^{n} \pi_{HS_j}}{\pi_{LN} + \sum_{j=1}^{n} \pi_{LS_j}} = \frac{P_H^w Z H_N + \sum_{j=1}^{n} P_H^w Z H_{S_j}}{P_L^w L_N + \sum_{j=1}^{n} P_L^w L_{S_j}} = \tilde{P}^w Z \tilde{h}^w,$$

where $\tilde{h}^w \equiv \left(H_N + \sum_{j=1}^{n} H_{S_j}\right) / \left(L_N + \sum_{j=1}^{n} L_{S_j}\right)$. This yields $\tilde{P}^w = \left(Z\tilde{h}^w\right)^{-1}$. Then, using (1.23), one obtains that $\tilde{A}_N = \tilde{A}_S = \tilde{A}_N^{IPR} \equiv \left(Z\tilde{h}^w\right)^{\epsilon-1}$, and $\tilde{w}_N = \tilde{w}_S = \tilde{w} = Z^{\epsilon-1}\left(\tilde{h}^w\right)^{\epsilon-2}$. That is, there is factor price equalization and both $\tilde{A}_N^{IPR}$ and $\tilde{w}_N$ are now smaller. Moreover, for given Z, the skill premium may even turn negative. To prevent this unreasonable outcome, we assume that skilled workers can take unskilled jobs and that a skilled worker produces Z times as much as an unskilled worker regardless of the sector of employment. This implies that there is a lower bound $\tilde{w} \geq Z$. When this lower bound is binding, the allocation of workers across the two sectors adjusts in order to keep $\tilde{w} = Z$. This leads to the following Proposition.

Proposition 3 *Assume free international trade in the intermediate goods Y_H and Y_L and IPR protection (licensing) in the South. For any $S \in \hat{S}$, the steady-state output ratio relative to the frontier is:*

$$\frac{Y_S}{Y_N} = \left(\frac{K_S}{K_N}\right)^{\alpha} \left(\frac{L_S + \tilde{w} H_S}{L_N + \tilde{w} H_N}\right)^{1-\alpha} \equiv f_S^{IPR}, \tag{1.33}$$

where $K_S/K_N = (Y_S/Y_N)/(\chi_S/\chi_N)$, $\tilde{h}^w = \left(H + \sum_{j=1}^{n} H_{S_j}\right) / \left(L + \sum_{j=1}^{n} L_{S_j}\right)$ and $\tilde{w} = \max\left\{Z^{\epsilon-1}(\tilde{h}^w)^{\epsilon-2},\ Z\right\}$.

Proof: The argument is parallel to the proof of Proposition 2. When $\tilde{w} > Z$, one obtains the analogue of expression (1.32),

$$\frac{Y_S}{Y_N} = \left(\frac{K_S}{K_N}\right)^{\alpha} \left(\frac{L_S}{L_N} \frac{1 + \tilde{P}^w \tilde{A}_N Z \tilde{h}_S}{1 + \tilde{P}^w \tilde{A}_N Z \tilde{h}_N}\right)^{1-\alpha}, \tag{1.34}$$

where the only differences between (1.32) and (1.34) is that in the latter $A_{LS} = A_{LN}$ and $\tilde{A}_N = \tilde{A}_S$. Next, substituting to $\tilde{P}^w$ and $\tilde{A}_N$ the respective expressions (i.e., $\tilde{P}^w = \left(Z\tilde{h}^w\right)^{-1}$ and $\tilde{A}_N = \left(Z\tilde{h}^w\right)^{\epsilon-1}$), and rearranging terms, leads to (1.33). When $\tilde{w} = Z$, a similar argument applies after noticing that:

$$\frac{P_H^w \hat{Y}_{HJ} + P_L^w \hat{Y}_{LJ}}{P_H^w \hat{Y}_{HN} + P_L^w \hat{Y}_{LN}} = \frac{w_H^w H_J + w_L^w L_J}{w_H^w H_N + w_L^w L_N} = \frac{Z H_J + L_J}{Z H_N + L_N}.$$

■

Cross-country productivity differences are smaller under full IPR. However, it becomes important to draw a distinction between GDP and (Gross National Product) GNP: the GNP of the North now includes the royalties paid by Southern firms. In general, it is ambiguous whether the GNP ratio increases with IPR. The growth rate of the world economy is unambiguously larger.

1.4 Empirical Analysis

In this section, we provide a quantitative assessment of the theory. The strategy is to use the no-trade economy of Section 1.2 as the benchmark for a development accounting exercise. More precisely, we consider a relative production function of the form:

$$\frac{y_S}{y_N} = \frac{\Omega_S}{\Omega_N} \times \frac{H_N + L_N}{H_S + L_S} \times f_S^{AUT}, \tag{1.35}$$

where f_S^{AUT} is given by (1.15).[6] Equation (1.35) allows for exogenous Hicks-neutral TFP differences (i.e., the term Ω_S/Ω_N) that are alien to our theory. Therefore, the success of our theory is measured by the extent to which the empirical variation in output and productivity can be accounted for without resorting to differences in Ω.

In the spirit of the development accounting literature (e.g., Caselli, 2005), we calibrate the key parameters, whenever this is possible. In particular, we set $\alpha = 0.35$ to match the non-labor share of GDP in industrialized countries, calibrate ϵ and Z so as to match the time evolution of the skill premium in the frontier economy using the predictions of our theory, and estimate ξ so as to obtain the best fit of cross-country productivity differences in two cross-sections of up to 122 countries (see Section 1.4.4 for more discussion). As it is customary, we use the no-trade scenario as the baseline case, and assess how successfully the benchmark model can account for the cross-country productivity distribution in 1970 and 2000. Then, we perform a number of theory-based counterfactuals including: (i) slashing all barriers to technology adoption, (ii) opening up the world economy to free trade, and (iii) allowing, in addition, perfect international IPR enforcement. We study the changes in the long-run distribution of productivity differences that each of these changes would trigger.

[6] Recall that, although the countries use different technologies, our theory is consistent with a common representation of the aggregate CES production function featuring increasing returns to scale.

1.4.1 Data Description

Since our analysis focuses on balanced-growth equilibria, we do not attempt to fit high-frequency data, and focus on the distribution of cross-country productivity differences in 1970 and 2000. We assume the US to be the frontier economy, and calibrate ϵ and Z using the change in the skill premium and the skill ratio between 1970 and 2000 in the United States from the March Current Population Survey (CPS) cleaned by Autor, Katz, and Kearney (2008).[7] Like these authors, we only consider full-time, full-year workers aged 16 to 64 with 0 to 39 years of potential experience. We exclude female workers and workers with earnings below $67 per week in 1982 dollars, as well as workers with allocated earnings. We calculate relative wages as the ratio of the CPS sampling weighted average earnings for different education levels. In particular, we focus on high school graduates vs. non-high school graduates and college graduates vs. non-college graduates.

The data on output, investment, population and the labor force are from Heston, Summers and Aten (2009). The estimates of the capital stock are generated using the perpetual inventory method (see, e.g., Caselli (2005)). For the relative skill endowment, we use two data sets: Barro and Lee (2010) and Cohen and Soto (2007). These data sets contain information on the fraction of the population aged 25 and above with a high school or a college degree. The stock of skilled and unskilled workers is then simply calculated by multiplying the labor force with the corresponding skill fraction in the population. Following Hall and Jones (1999), we perform a natural resource correction on GDP by subtracting the fraction of value added in the mining and quarrying sector according to National Accounts Official Country Data accessed via UNdata. Because for some countries value added in the mining and quarrying sector is not reported on an annual basis, we linearly interpolate the missing data points for 1970 and 2000 if necessary. We drop Kuwait which is a strong outlier in terms of GDP pw in 1970.[8] We end up with a repeated cross-section of 86 (1970) to 122 (2000) countries when using the education data from Barro and Lee (2010), while we have 73 (1970) to 85 (2000) countries when using the data from Cohen and Soto (2007). In Appendix A we also we repeat the analysis restricting the balanced sample of countries for which information is available both in 1970 and 2000 and find very similar results.

[7] In practice, we use the observations of 1971 and 2001 since the reported earnings are for the previous year. The two data sets are available online from David H. Autor's website.

[8] The inclusion of Kuwait does not change our main results. In Table A.3 of Appendix A we report the estimation results for a sample that includes Kuwait.

1.4.2 Calibration

Elasticity of Substitution

We identify ϵ and Z using equation (1.11) given the evolution of the skill premium in the US. More formally, we set ϵ and Z so as to match exactly the equation:

$$\log\left(\tilde{w}_{US,t}\right) = (\epsilon - 1)\log(Z) + (\epsilon - 2)\log\left(\tilde{h}_{US,t}\right), \tag{1.36}$$

where $t \in \{1970, 2000\}$. Hence

$$\epsilon = 2 + \frac{\log\left(\tilde{w}_{US,2000}\right) - \log\left(\tilde{w}_{US,1970}\right)}{\log\left(\tilde{h}_{US,2000}\right) - \log\left(\tilde{h}_{US,1970}\right)}. \tag{1.37}$$

The skill premia as well as the skill ratios are taken from the March CPS. As discussed above, we use two alternative measures of skill: secondary and tertiary school. The wage premium for high school graduates over non-high school graduates increased from 1.40 in 1970 to 2.02 in 2000, while the wage premium for college graduates over non-college graduates increased from 1.57 in 1970 to 1.88 in 2000. The ratio of high school graduates over non-high school graduates in the population in working age increased during the same period from 2.59 to 9.30, while the ratio of college graduates over non-college graduates increased from 0.21 to 0.43. Since in many OECD economies a large share of the population finishes secondary school, we regard tertiary education as the most appropriate measure of skill for our theory.

Table 1.1: Baseline calibration

Skill measure	ϵ	Z
sec	2.29	1.05
tert	2.25	1.96

Equations (1.36)-(1.37) pin down ϵ and Z. Since both the skill ratio and the relative skill supply increased sharply in the United States during 1970–2000, the two equations imply that $\epsilon > 2$. Table 1.1 summarizes the baseline calibration for ϵ and Z conditional on the skill measure. In the table (and for future reference), *sec* stands for "secondary school completed" whereas *tert* stands for "tertiary school completed".

In our model, the parameter ϵ has the structural interpretation of a short-run elasticity between high- and low-skill labor. Other studies (e.g., Ciccone and Peri (2005)) provide estimates of such an elasticity of substitution in the interval $[1.4, 2]$. Since our estimate of ϵ falls outside of this range, we consider lower values in Section 1.4.4. Note that if we calibrate ϵ to lower values, we must allow Z to increase between 1970 and 2000, or else the theory would predict, counterfactually, a decline in the skill premium. In other words, our estimate $\epsilon > 2$ appears to be consistent with the prediction of our theory, whereas lower values of ϵ are rejected by our estimation unless we assume that there are other exogenous drivers of skill-biased technical change, captured by an increase in Z.

Barriers to Technology Adoption

Having calibrated α, Z and ϵ as described above, we estimate ξ by full information maximum likelihood (FIML) using the following econometric model:

$$\log\left(\frac{y_S}{y_{US}}\right) = \log\left[f_S^{AUT} \times \frac{H_N + L_N}{H_S + L_S}\right] + \log\varepsilon_S,$$

where f_S^{AUT} is given by (1.15), and $\log\varepsilon_S$ is an i.i.d. normally distributed disturbance with mean zero.

Table 1.2 shows the estimation results with robust standard errors in parentheses. The four rows refer to different skill categories (*sec* and *tert*) and data sets (Barro-Lee (BL) and Cohen-Soto (CS)). Columns 1 and 2 report the point estimate of ξ using the whole sample. Then, we allow ξ to vary between OECD (columns 3 and 4) and non-OECD countries (columns 5 and 6). The results show that ξ is significantly lower in non-OECD countries,[9] which is consistent with the interpretation that poor countries have larger barriers. Since there remains a great deal of heterogeneity within non-OECD countries, we split further the subsample into sub-Saharan (columns 7 and 8) and other non-OECD countries (columns 9 and 10).[10] The differences in barriers to technology adoption between both the sub-Saharan and other non-OECD countries and OECD and non-OECD countries are in all but one cases highly significant.[11]

[9] We classify as OECD all countries that were OECD members in 2000 (same classification in both 1970 and 2000 to limit endogeneity issues). Including only countries that were OECD members in 1970 yields similar results. The estimates for OECD countries are then higher while those for non-OECD countries remain almost unchanged. For instance, the point estimate for OECD countries in the third row of Table 1.2 would be 6.61 (1.80) in 1970 and 21.50 (10.26) in 2000 instead of 5.91 (1.48) in 1970 and 11.71 (3.37) in 2000.

[10] We do not include Mauritius among the sub-Saharan countries, due to its special geographical and economic conditions (see Subramanian and Roy 2001). Including Mauritius would not cause dramatic changes in the point estimates. For instance, the third row in Table 1.2 would read 2.33 (0.20) in 1970 and 2.52 (0.30) for sub-Saharan countries and 3.83 (0.48) in 1970 and 3.94 (0.45) in 2000 for the other non-OECD countries.

[11] In 1970, the point estimate for sub-Saharan countries is lower than the point estimate for the other non-OECD countries at the 1 percent level of significance across all specifications. In 2000, barriers are significantly lower for the *tert* skill category at the 1 percent (BL) and 5 percent level (CS). For the *sec* skill category, the differences are significant at the 5 percent for BL and close to the 10 percent level of significance for CS. OECD countries have significantly lower barriers than non-OECD countries at the 1 percent level in 2000 for the *tert* skill category, while they are at least lower at the 5 percent level of significance for the *sec* skill category and in 1970 across all specifications.

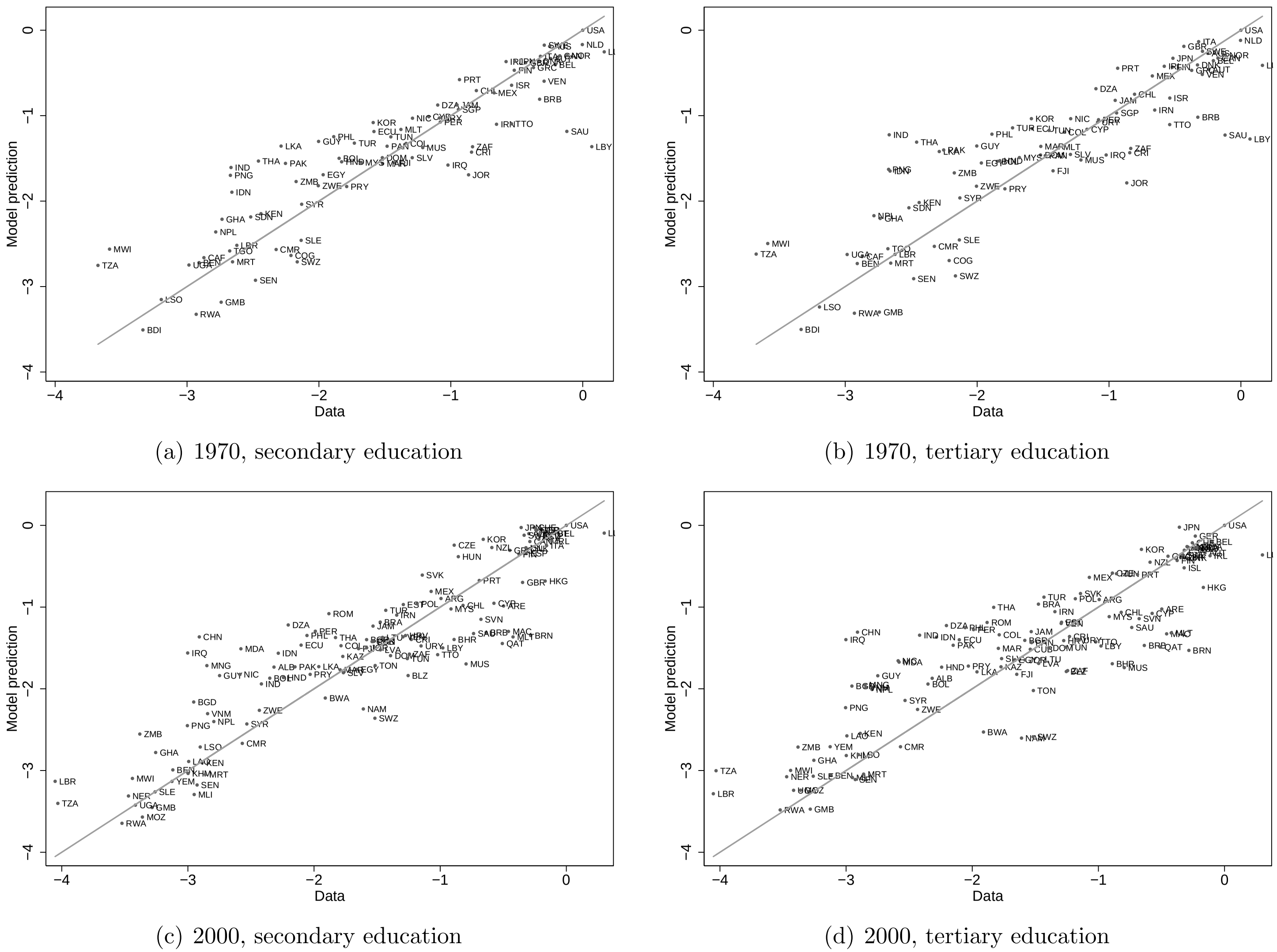
Table 1.2: Baseline estimation

		All countries		OECD		Non-OECD					
						All		Sub-Sahara		Others	
		1970	2000	1970	2000	1970	2000	1970	2000	1970	2000
Data	Skill	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
BL	sec	4.84	5.38	10.72	18.66	4.51	4.80	3.17	3.80	6.13	5.33
		(0.50)	(0.54)	(3.60)	(8.27)	(0.47)	(0.49)	(0.31)	(0.47)	(1.07)	(0.72)
CS	sec	3.78	3.98	6.40	11.32	3.75	3.48	2.25	3.00	4.90	3.77
		(0.35)	(0.38)	(1.09)	(2.47)	(0.35)	(0.34)	(0.24)	(0.32)	(0.60)	(0.52)
BL	tert	3.19	3.78	5.91	11.71	3.02	3.38	2.25	2.35	3.88	4.05
		(0.25)	(0.33)	(1.48)	(3.37)	(0.25)	(0.30)	(0.19)	(0.24)	(0.47)	(0.47)
CS	tert	3.23	2.83	5.53	8.41	3.05	2.46	1.97	1.86	4.14	2.92
		(0.28)	(0.24)	(0.96)	(1.37)	(0.28)	(0.21)	(0.19)	(0.13)	(0.45)	(0.36)
Obs.	(BL/CS)	85/71	121/83	19/17	29/23	66/54	92/60	23/19	23/19	43/35	69/41

Another pattern emerges from the table: The estimated ξ approximately doubles between 1970–2000 for OECD countries, while there is no big change for non-OECD countries. This suggests that technological integration increased mostly within the set of industrialized countries.

Results

Figure 1.2 plots the predicted GDP pw (log-difference from the US) against the actual GDP pw for all countries, using educational variables from the Barro-Lee data set and allowing ξ to differ across OECD, sub-Saharan and other non-OECD countries, as in Table 1.2. Panels (a)-(c) use the secondary school educational measure for years 1970 and 2000, respectively, whereas Panels (b)-(d) use the tertiary education measure for the same years. In Appendix A we plot the corresponding figure that is obtained by imposing a common ξ over the entire sample. Whenever a point lies on the 45-degree line, the theory fits the data perfectly. Whenever a point lies above (below) the 45-degree line, the model underpredicts (overpredicts) the productivity differences between that country and the US. The fit is altogether good, although there are some outliers, among them, Malta, Cyprus and Hong-Kong which lie significantly below the 45-degree line. This is not surprising, since these countries are classified as non-OECD countries (and thus pooled in the estimation of ξ with poorer economies), although they are very open economies sharing more commonalities with the OECD countries than with the rest of non-OECD countries. Since the estimation forces them to have large barriers, the model largely overpredicts their productivity difference relative to the US. If one merges these three countries with the OECD, they cease to be outliers. Likewise, Bahrain, Barbados, Brunei, Mauritius and Qatar (also below the



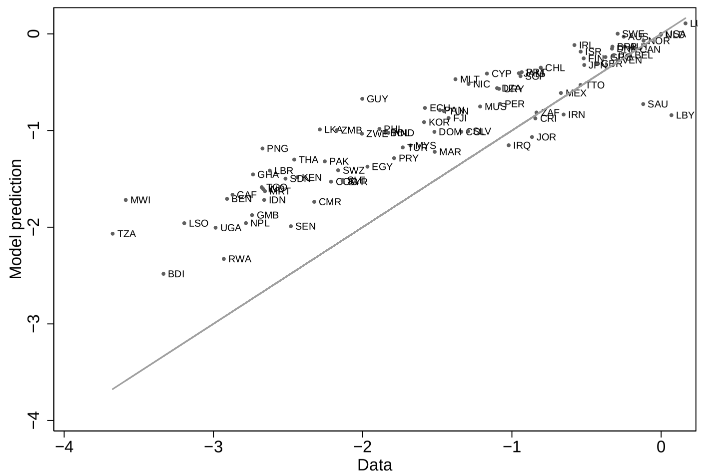
(a) 1970, secondary education

(b) 1970, tertiary education

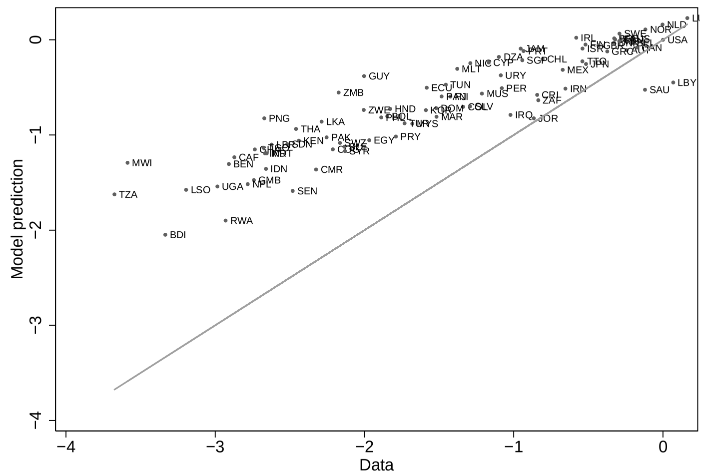
(c) 2000, secondary education

(d) 2000, tertiary education

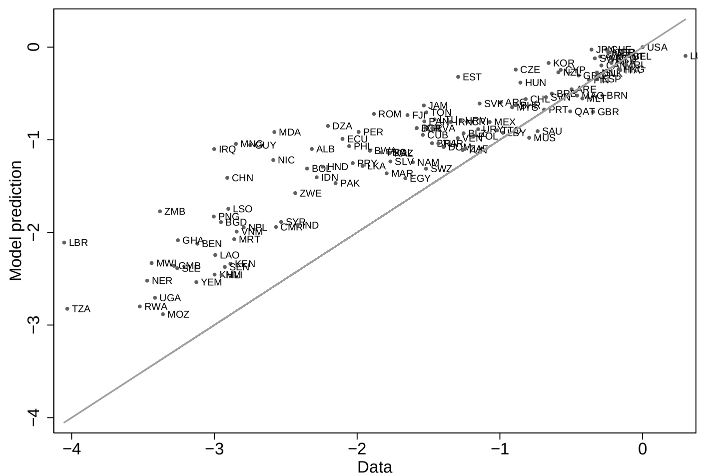
Figure 1.2: Baseline estimation: GDP pw (log-difference from the US).



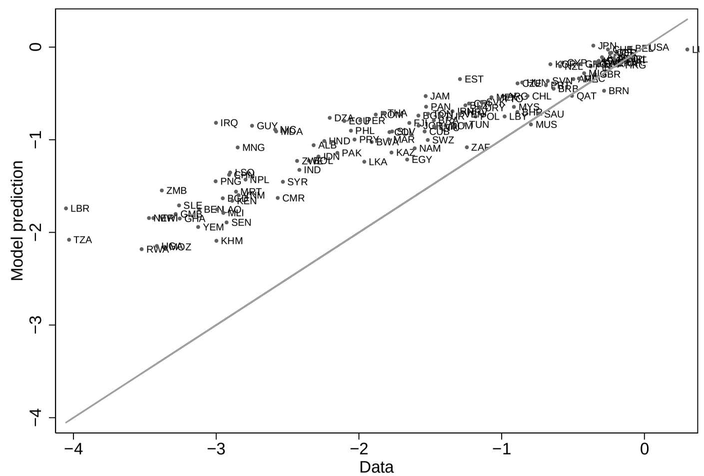
(a) 1970, secondary education



(b) 1970, tertiary education



(c) 2000, secondary education



(d) 2000, tertiary education

Figure 1.3: No barriers to technology adoption: GDP pw (log-difference from the US).

45-degree line) are small economies with special characteristics that make them atypical non-OECD economies. Among the countries lying significantly above the 45-degree line, one notices Japan, Korea and China in year 2000. The large population size and/or the high physical capital per worker are behind this finding.

It is useful to compare the results with those that would obtain if we estimated productivity differences assuming no barriers to technology adoption, as in AZ01. More formally, we let $\xi \to \infty$ – see equation (1.22) – while keeping all other parameters unchanged. AZ01 find that their model yields a significantly better fit than a neoclassical one-sector model such as the one used by Hall and Jones (1999). Since our model encompasses their specification as a particular case, we can quantify the importance of barriers, separating their effect from that of "inappropriate technology". Figure 1.3 is the analogue of Figure 1.2 but letting $\xi \to \infty$. It shows that the model without barriers underestimates significantly the cross-country productivity differences.

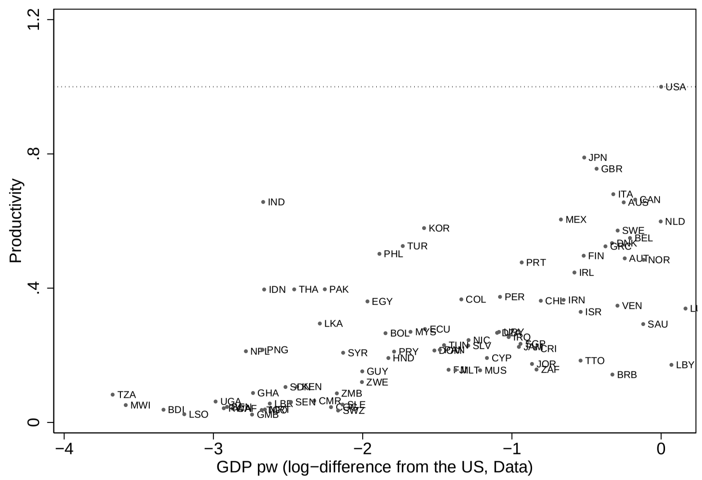
To compare the goodness of fit of the two models more formally, we use the statistic proposed by AZ01:

$$\Re^2 = 1 - \sum_{j=1}^{n} \left(\log(y_{S_j}/y_{US}) - \widehat{\log}(y_{S_j}/y_{US}) \right)^2 / \sum_{j=1}^{n} \left(\log(y_{S_j}/y_{US}) \right)^2 ,$$

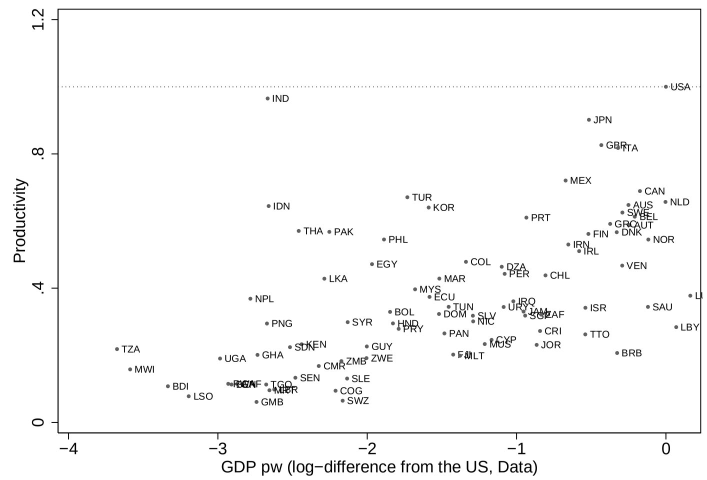
where $\log(y_{S_j}/y_{US})$ denotes the log-difference in output per worker from the US in the data and $\widehat{\log}(y_{S_j}/y_{US})$ the prediction of the model for the same country. $\Re^2$ would be equal to 1, if all points were aligned on the 45-degree line. In this case, the model would fit the data perfectly. Note that $\Re^2$ is not a standard R-squared, and can be negative if the fit is sufficiently low. Table 1.3 reports the $\Re^2$ for the three specifications of Table 1.2, and for comparison also the case of no barriers (column 4). In column 1 all countries are constrained to have the same ξ. In column 2 ξ is allowed to differ between OECD and non-OECD countries. Finally, in column 3, we also allow ξ to differ between sub-Saharan and other non-OECD countries. In all cases, the model with barriers attains a much better fit than the model with no barriers.[12] The model with no barriers is also rejected in a formal Wald test.

A concern is that our estimation may imply $A_{LS}/A_{L,US}$ and/or $A_{HS}/A_{H,US}$ larger than unity, violating the assumption that the US is the technology leader in both sectors. To address this concern, Figure 1.4 plots the implied cross-country distribution of the sectoral productivities, $A_{LS}/A_{L,US}$ and $A_{HS}/A_{H,US}$, using our estimate of the baseline model in the

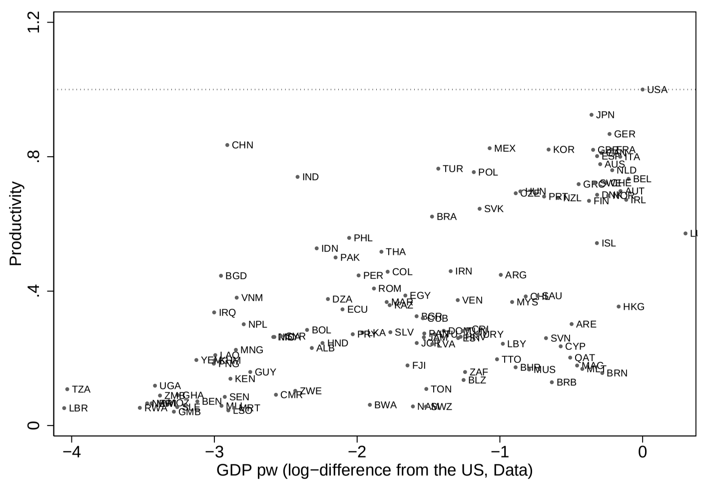
[12] The results are not directly comparable with those of AZ01. First their model implies $\epsilon = 2$, and they set $Z \in \{1.5, 1.8\}$ to match the skill premium. Second, they use data for 1990. To make the comparison more direct, we re-estimated our model after calibrating $\epsilon = 2$ and $Z = 1.8$, using the two educational measures from BL for the year of 2000. The $\Re^2$ of the model without barriers is 0.871 and 0.766 using *sec* and *tert*, respectively. In contrast, the $\Re^2$ of column 3 in Table 1.3 would be 0.940 and 0.922, respectively.



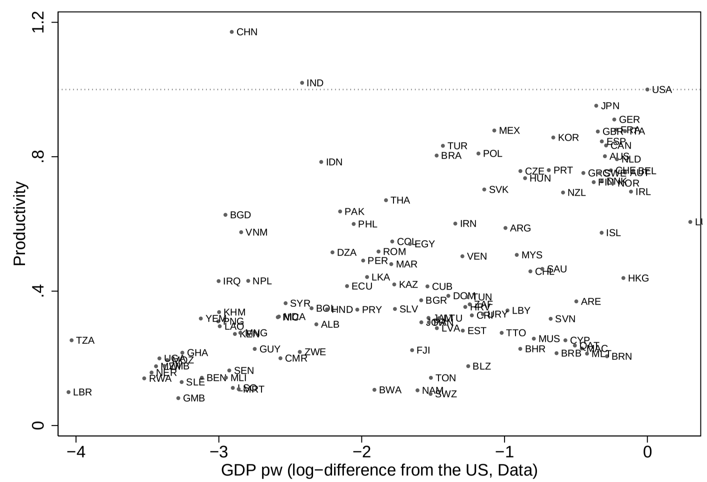
(a) 1970, high-skill sector



(b) 1970, low-skill sector



(c) 2000, high-skill sector



(d) 2000, low-skill sector

Figure 1.4: Sectoral productivities (relative to the US).

Table 1.3: Goodness of fit

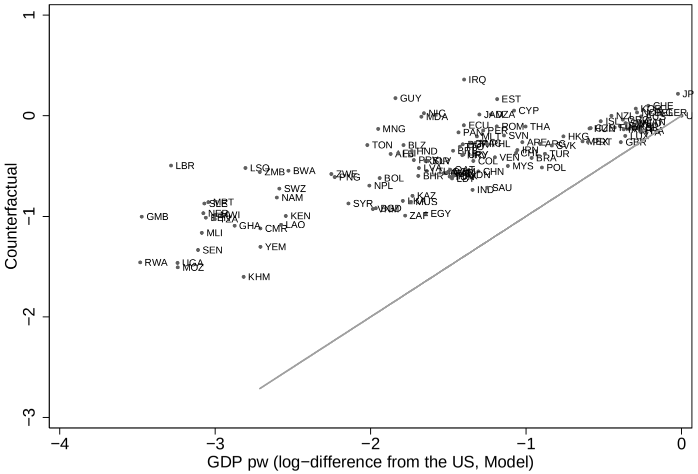
		Baseline estimation						No barriers	
		(1)		(2)		(3)		(4)	
Data	Skill	1970	2000	1970	2000	1970	2000	1970	2000
BL	sec	0.926	0.930	0.929	0.936	0.938	0.938	0.829	0.855
CS	sec	0.934	0.944	0.937	0.951	0.953	0.952	0.807	0.856
BL	tert	0.905	0.903	0.910	0.913	0.923	0.921	0.690	0.763
CS	tert	0.924	0.922	0.927	0.936	0.947	0.942	0.746	0.761

case of tertiary education with BL data. The hypothesis that the US is the technology leader is never rejected in the skilled sector. More formally, $A_{HS}/A_{H,US} < 1$ for all S. This is not surprising. More interesting, the hypothesis that the US is the technology leader in the low-skill sector is only contradicted in the case of China and India in 2000. This is due to the large market for low-skill technologies available in those two countries. Since it seems empirically implausible that China and India use all technologies currently in use in the US in the low-skill sector, this finding suggests that the model may exaggerate the role of market size effect in technology adoption. Alternatively, the assumption that large developing economies such as China and India have frictionless internal markets may be incorrect. Altogether, we find it reassuring that – with only two (important) exceptions – the assumption that the US is the leader is consistent with our estimation, without the need of imposing any additional restriction.

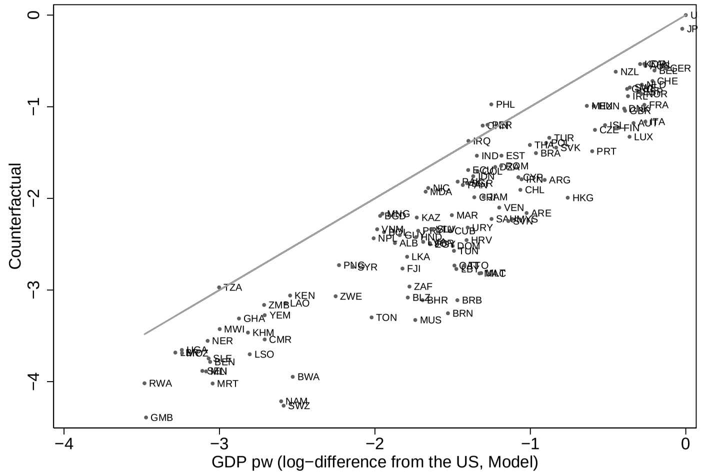
1.4.3 Counterfactuals

In this section we use our model as a lab to perform three counterfactual experiments. We assume the economies to be initially in the no-trade steady state of year 2000, and study the long-run effect of institutional changes on cross-country inequality. The three experiments consist of, respectively: (i) removing all barriers to technology adoption, (ii) opening up the world economy to frictionless international trade, and (iii) introducing, in addition, full international IPR enforcement. We focus on steady-state effects.

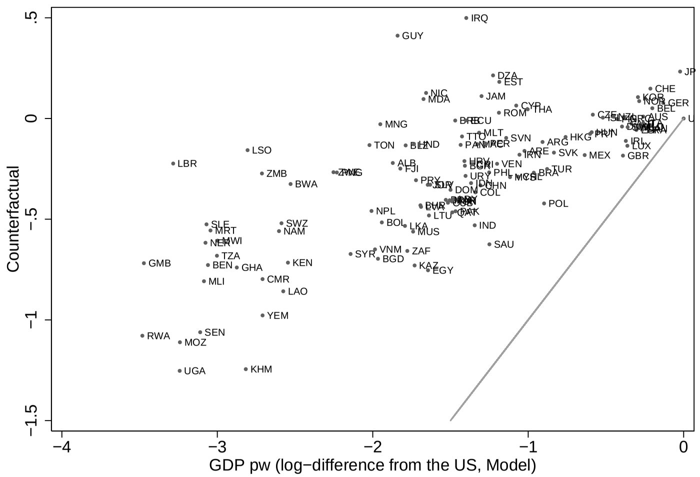
We limit our discussion to the *tert* skill measure from the Barro-Lee data set and to the case in which ξ differs between OECD, sub-Saharan and other non-OECD countries (column 3 to 4 and 7 to 10 in Table 1.2). The parameters α, Z, ϵ and ξ are held constant across experiments at the levels of Section 1.4.2 (with the exception of experiment (i) when we let $\xi \to \infty$). Since physical capital is endogenous, we allow the capital-output ratio to respond to institutional changes. We do so by first inferring from the observed capital-output ratios the cross-country distribution of the deep parameter χ (the "investment wedge") in the benchmark no-trade case. Next, we calculate the capital-output ratio that



(a) No barriers to technology adoption



(b) Free trade



(c) Free trade and perfect IPR protection

Figure 1.5: Counterfactual GDP pw (log-difference from the US).

would obtain in each of the counterfactual steady states (no barriers, free trade and trade with full IPR enforcement) assuming no change in χ. Since our target is to estimate relative productivities, we focus on the distribution of investment wedges relative to the North. For country S such ratio is given by:

$$\frac{\chi_S}{\chi_N} = \frac{Y_S/K_S}{Y_N/K_N}, \tag{1.38}$$

where the right hand-side term is the capital-output ratio in the data, and we continue to assume the US to be the frontier economy. Next, letting variables indexed by the superscript $count \in \{nobarr, trade, IPR\}$ denote theoretical steady-state levels in each counterfactual, we obtain:

$$\frac{K_S^{count}}{K_N^{count}} = \frac{Y_S^{count}/\chi_S}{Y_N^{count}/\chi_N} = \frac{Y_S^{count}}{Y_N^{count}}\frac{K_S/Y_S}{K_N/Y_N}. \tag{1.39}$$

Replacing K_S/K_N by $K_S^{nobarr}/K_N^{nobarr}$, K_S^{trade}/K_N^{trade} and K_S^{IPR}/K_N^{IPR}, respectively, into equations (1.22), (1.29) and (1.33), and rearranging terms, yields the steady-state expressions for output and productivity reported in each of the subsections below.

No Barriers

In this section, we experiment with slashing all technology barriers. Such experiment differs from the analysis in Section 1.4.2, as there we treated the no-barrier model as an alternative model and estimated equation (1.22) taking the capital ratio directly from the data. In contrast, here we infer the χ from the benchmark case and let capital adjust in each country to the new steady state, as discussed above. The gains in output per worker will be larger for countries with smaller investment wedges, since slashing barriers induces a stronger increase in investments in physical capital in those countries.

We obtain the following counterfactual steady-state output gaps:

$$\begin{aligned}
\frac{Y_S^{nobarr}}{Y_N^{nobarr}} &= \left(\frac{K_S^{nobarr}}{K_N^{nobarr}}\right)^{\alpha} \times \left[\frac{L_S^{\frac{\epsilon-1}{\epsilon}} + \left(Z\tilde{h}_N\right)^{\frac{(\epsilon-1)^2}{\epsilon}} \times (ZH_S)^{\frac{\epsilon-1}{\epsilon}}}{L_N^{\frac{\epsilon-1}{\epsilon}} + \left(Z\tilde{h}_N\right)^{\frac{(\epsilon-1)^2}{\epsilon}} \times (ZH_N)^{\frac{\epsilon-1}{\epsilon}}}\right]^{\frac{\epsilon(1-\alpha)}{\epsilon-1}} \\
&= \left(\frac{K_S/Y_S}{K_N/Y_N}\right)^{\frac{\alpha}{1-\alpha}} \times \left[\frac{L_S^{\frac{\epsilon-1}{\epsilon}} + \left(Z\tilde{h}_N\right)^{\frac{(\epsilon-1)^2}{\epsilon}} \times (ZH_S)^{\frac{\epsilon-1}{\epsilon}}}{L_N^{\frac{\epsilon-1}{\epsilon}} + \left(Z\tilde{h}_N\right)^{\frac{(\epsilon-1)^2}{\epsilon}} \times (ZH_N)^{\frac{\epsilon-1}{\epsilon}}}\right]^{\frac{\epsilon}{\epsilon-1}},
\end{aligned}$$

where K_S/Y_S and K_N/Y_N are the observed capital output ratios. Panel (a) in Figure 1.5

plots the counterfactual log GDP pw relative to the US, $\widehat{\log}(y_S^{nobarr}/y_{US}^{nobarr})$, against the productivity differences predicted by the benchmark model. There are significant gains for most countries, which are especially large for those with small investment wedges. Among the OECD economies making largest gains, one notices New Zealand, Korea, Hungary, Czech Repulic, Slovak Republic and Switzerland. On average, the GDP pw relative to the US improves for an OECD country from 0.68 to 0.91 while non-OECD countries increase from 0.19 to 0.61. The effect is particularly strong for small countries, which lack the local market size required to benefit from expensive technologies (for instance, Cyprus improves from 0.34 to 1.05 while the United Kingdom only increases from 0.68 to 0.77).

Trade

In this section we consider the effects of opening up the world economy to free trade. The counterfactual steady-state output differences are given by equation (1.29), after replacing K_S/K_N by K_S^{trade}/K_N^{trade}, as given by equation (1.39). This yields:

$$\frac{Y_S^{trade}}{Y_N^{trade}} = \left(\frac{K_S/Y_S}{K_N/Y_N}\right)^{\frac{\alpha}{1-\alpha}} \times \frac{L_S^{\frac{1+\xi}{\xi}} + \frac{\left(Z\tilde{h}_N\right)^{\frac{\xi(\varepsilon-1)-(1+\xi)}{\xi}}}{\hat{h}} \times (ZH_S)^{\frac{1+\xi}{\xi}}}{L_N^{\frac{1+\xi}{\xi}} + \frac{\left(Z\tilde{h}_N\right)^{\frac{\xi(\varepsilon-1)-(1+\xi)}{\xi}}}{\hat{h}} \times (ZH_N)^{\frac{1+\xi}{\xi}}}.$$

As discussed in Section 1.3.1, trade increases the skill bias of the frontier technology, while its effect on the skill bias of technology adoption is ambiguous.

Panel (b) in Figure 1.5 plots $\widehat{\log}(y_S^{trade}/y_{US}^{trade})$ against the predictions of the benchmark model. Cross-country income inequality increases significantly, and so does the distance of most countries from the US frontier. The GDP pw relative to the US decreases for the average OECD country from 0.68 to 0.41, while the non-OECD countries fall from 0.19 to 0.10. Among the OECD countries that realize the largest losses are Italy, Luxembourg, Austria, France, and Finland. However, it is also important to remind that trade implies an increase in the growth rate of all economies, so a loss in relative terms does not imply a welfare loss.

Trade and IPR

In this section, we focus on trade with perfect IPR protection, following the theoretical analysis of Section 1.3.2. The counterfactual steady-state output differences are given by equation (1.33), after replacing K_S/K_N by K_S^{IPR}/K_N^{IPR} as given by (1.39). This yields:

$$\frac{Y_S^{IPR}}{Y_N^{IPR}} = \left(\frac{K_S/Y_S}{K_N/Y_N}\right)^{\frac{\alpha}{1-\alpha}} \times \frac{L_S + \tilde{w}H_S}{L_N + \tilde{w}H_N},$$

where $\tilde{w} = \max\left\{Z^{\epsilon-1}(\tilde{h}^w)^{\epsilon-2},\ Z\right\}$. As discussed in Section 1.3.2, all countries use now the frontier technology, as in the case of no barriers. However, the frontier technology is now less skill biased. Panel (c) in Figure 1.5 plots $\widehat{\log}(y_S^{IPR}/y_{US}^{IPR})$ against the productivity differences predicted by the benchmark model. The results are similar to those in panel (a), but the relative gains of non-frontier economies are larger. Many economies – including most European countries – would now surpass the US. The reason is twofold. First, the skill bias of the technology targets the average world endowment so innovation is too little skill biased for the most skilled rich countries such as the US. Second, many countries have a higher capital output ratio than the US. However, it is important to remember that non-frontier countries must transfer to the US a significant share of their GDP as license fees. So, the differences in GNP may be significantly larger than the differences in GDP.

Overall, these results are in line with AZ01 and Bonfiglioli and Gancia (2008), who show in more specific models that trade opening with no global IPR protection may induce a wave of technological progress which favors disproportionately the North, while stronger IPR protection in the South can speed up technology transfer and reduce income differences.

Wage Inequality

Finally, we consider the prediction of the theory for the changes in wage inequality in the three counterfactual scenarios relative to the benchmark case. Recall $\tilde{w}_S = Z \cdot \tilde{P}_S \cdot \tilde{A}_S$. In autarky, $\tilde{P}_S$ and $\tilde{A}_S$ are given by (1.7) and (1.14), respectively. The same expressions hold with no barriers to adoption after letting $\xi \to \infty$. In the free-trade case, prices are equalized worldwide to $\tilde{P}^w = \left(Z\tilde{h}_N\right)^{-1}$ and $\tilde{A}_S = \hat{h}^{-1}\left(Z\tilde{h}_N\right)^{\epsilon-1}\left(\tilde{h}_S/\tilde{h}_N\right)^{1/\xi}$, where $\hat{h}$ is given by (1.25). Finally, in the case of trade with IPR, we have $\tilde{w}_S = \max\left\{Z^{\epsilon-1}(\tilde{h}^w)^{\epsilon-2},\ Z\right\}$, where $\tilde{h}_w$ is the world average relative skill endowment.

Figure 1.6 plots the log-change in the steady-state skill premium for tertiary school against GDP per worker relative to the US when barriers are removed starting from the benchmark steady state equilibrium. Removing barriers implies an increase in the skill premia of non-frontier economies, since costly adoption reduces the skill bias of the technology adoption. The effect is stronger the farther away from the frontier a country is. For the average non-OECD country the skill premium increases by 25 percent, while it rises only by 3 percent among OECD countries. Figure 1.7 plots the corresponding log-change in the steady-state skill premium when an economy switches to free trade. Opening up to free trade in goods raises the skill premium in skill-abundant countries and lowers it in skill-scarce countries, as predicted by the Stolper-Samuelson theorem. However, by also inducing skill-biased technical change at the frontier, it generates an upward pressure on the skill premium worldwide. As a result, wage inequality increases in the majority of

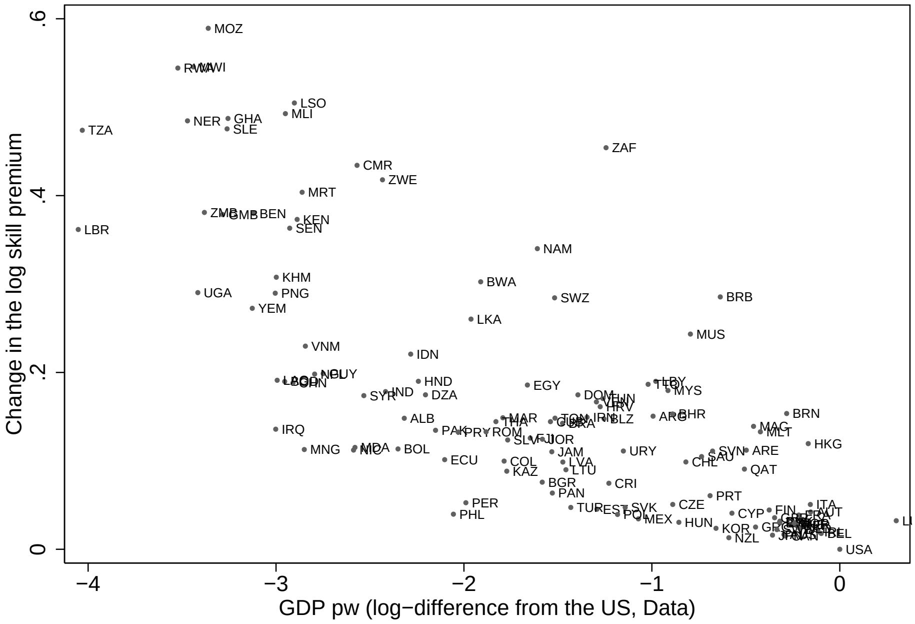


Figure 1.6: Change in skill premium: benchmark to no barrier counterfactual.

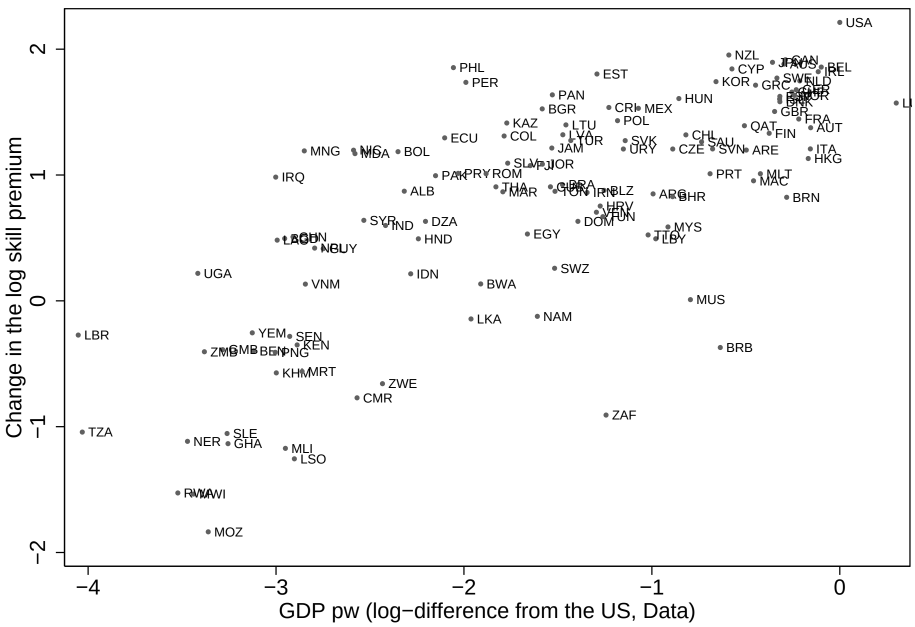


Figure 1.7: Change in skill premium: benchmark to free trade counterfactual.

countries, particularly in skill-abundant and low-barriers countries. The conventional result that trade liberalization lowers inequality in skill-scarce countries holds only in the group of economies facing the highest barriers to technology adoption (for instance, in sub-Saharan countries the skill premium falls on average by 42 percent), while wage inequality rises even in India and China.

Finally, when IPR are also protected (no figure), the relevant market for new technologies becomes the world economy. This promotes the development of low-skill technologies and thus a fall in the skill premium. Moreover, since all countries now use the same technologies, all wages become the same everywhere. Given the large endowment of unskilled labor of the world economy, we find that with trade and IPR protection $\tilde{A}$ falls so much that the constraint $\tilde{w}_S \geq Z$ becomes binding. Thus, in the new steady state wage inequality drops to $\tilde{w} = Z$ in all countries. Before concluding, it is important to emphasize that these large changes in skill premia reflect the rather extreme nature of our counterfactual scenarios. The effect of partial integration of the markets for goods and technology would certainly be smaller. It is also important to stress that our model abstracts from differences in labor market institutions and policies which are likely to affect the cross-country pattern of skill premia and its change under the alternative scenarios.

1.4.4 Robustness

In this section we analyze the robustness of our results. First, we study the robustness of the model to different calibrations of ϵ. Then, we compare our results with those that would obtain from an atheoretical development accounting exercise. Next, we test the robustness of the results to a weaker form of the market size effect. Last, we estimate the model under the alternative assumption that in year 2000 all economies are open to international trade.

Lower Short-Run Elasticity of Substitution

In this part, we study the robustness of our model to a different calibration of ϵ. Earlier studies find the short-run elasticity of substitution between skilled and unskilled labor to be in the range $\epsilon \in [1.5, 2]$. It is important to stress that $\epsilon < 2$ is inconsistent in our model with the observation of increasing skill premia in the US during 1970-2000. To reconcile lower ϵ's with the evolution of the skill premium in the US, we must then allow for an exogenous increase in Z. The new calibration is summarized in Table 1.4, where we restrict attention to *tert* from the Barro-Lee dataset which is our preferred measure of skill.

Table 1.5 shows the new estimates of ξ. When $\epsilon = 2$, the results are qualitative similar to those of the benchmark case, although the estimates of ξ are somewhat larger. The $\Re^2$ are still above 0.9, and the differences in ξ across groups and time remain at the significance

Table 1.4: Robustness calibration

	$\epsilon = 2$		$\epsilon = 1.5$	
Skill	Z_{1970}	Z_{2000}	Z_{1970}	Z_{2000}
tert	1.57	1.88	0.51	1.52

level of the baseline estimation in Table 1.2. In summary, our analysis is not affected by setting $\epsilon = 2$. When $\epsilon = 1.5$, the results continue to be similar to the benchmark case. The estimates go further up, and the level of significance reduces to 5 percent between OECD and non-OECD countries in 2000. In spite of this, the goodness of fit stays above 0.9.

Table 1.5: Robustness estimation

		All countries		OECD		Non-OECD					
						All		Sub-Sahara		Others	
		1970	2000	1970	2000	1970	2000	1970	2000	1970	2000
Data	Skill	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\epsilon = 2$											
BL	tert	3.31	3.90	6.17	12.09	3.13	3.50	2.33	2.47	4.03	4.16
		(0.27)	(0.35)	(1.57)	(3.54)	(0.26)	(0.31)	(0.19)	(0.25)	(0.50)	(0.49)
$\epsilon = 1.5$											
BL	tert	3.74	4.38	7.00	13.53	3.54	3.94	2.65	2.93	4.56	4.58
		(0.31)	(0.40)	(1.86)	(4.22)	(0.30)	(0.36)	(0.22)	(0.30)	(0.61)	(0.56)
Obs.		85	121	19	29	66	92	23	23	43	69

Alternative Specifications

A number of papers (discussed in the introduction) perform development accounting exercises based on reduced form aggregate production functions such as equation (1.1). The wisdom of this literature is that the model can replicate the empirical cross-country productivity distribution as long as one imposes sufficiently low elasticities of substitution between factors of production. For instance, Caselli (2005) shows that if one calibrates a production function with physical and human capital allowing for very low values of the elasticity of substitution, one can fit arbitrarily well the cross-country data. In this paper, we allow ourselves no freedom in the choice of the elasticity of substitution between capital and labor, which we taken to be unit as it is standard in the growth accounting literature. In addition, we estimate the short-run elasticity of substitution between high- and low-skill labor using the time-series implication of the theory. The only parameter on which we impose no *a priori* restriction is ξ. We should note, though, that our theory imposes that the *long-run* elasticity of substitution between high- and low-skill labor be larger than the short-term elasticity. Thus, estimating ξ does not imply a degree of freedom in the choice of the elasticity of substitution, and our theory precludes that a good fit can arise from low elasticities.

Nevertheless, it is interesting to compare the success of our theory with that of a reduced form production function approach.[13] For the sake of such comparison, we estimate the following alternative (reduced form) model:

$$\frac{Y_S}{Y_N} = \left(\frac{K_S}{K_N}\right)^{\alpha} \left(\frac{L_S^{\frac{\upsilon-1}{\upsilon}} + \left(\frac{\bar{A}_H}{\bar{A}_L} H_S\right)^{\frac{\upsilon-1}{\upsilon}}}{L^{\frac{\upsilon-1}{\upsilon}} + \left(\frac{\bar{A}_H}{\bar{A}_L} H_N\right)^{\frac{\upsilon-1}{\upsilon}}}\right)^{\frac{\upsilon(1-\alpha)}{\upsilon-1}}, \tag{1.40}$$

subject to the restriction that labor markets are competitive, implying that:

$$\frac{\bar{A}_H}{\bar{A}_L} = (\tilde{w}_{US})^{\frac{\upsilon}{\upsilon-1}} \left(\frac{H_N}{L_N}\right)^{\frac{1}{\upsilon-1}},$$

where $\tilde{w}_{US}$ is the observed skill premium in the US and $\upsilon \geq 0$ is the elasticity of substitution between low- and high-skill labor.

Consistent with previous studies, we find that the best fit of this model obtains with low elasticities of substitution between high- and low-skill workers. For instance, if we measure skill by tertiary school from the Barro-Lee data set the best estimates yield $\upsilon_{1970} = 1.07$ and $\upsilon_{2000} = 0.50$. With such low elasticities, the model fits quite well the data. In particular, we obtain $\Re^2_{1970} = 0.771$ and $\Re^2_{2000} = 0.916$. However, the estimated elasticities are clearly outside of the consensus range. If we impose that $\upsilon \geq 1.5$, the goodness of fit falls significantly. For instance, with tertiary education and $\upsilon = 1.5$ one obtains $\Re^2_{1970} = 0.726$ and $\Re^2_{2000} = 0.802$ with BL and $\Re^2_{1970} = 0.785$ and $\Re^2_{2000} = 0.800$ with CS. For comparison, the corresponding $\Re^2$s of Table 1.3 range between 0.903 and 0.952. In addition, the reduced form model systematically underpredicts the cross-country productivity differences for reasonable values of υ. On both grounds, the reduced form model performs significantly worse than our structural model with tertiary education. In sum, a reduced form model without market-size effects does not outperform our structural model.

[13] It is important to note that our model is *not* observationally equivalent to a standard aggregate constant returns to scale CES production function like (1.1) for two reasons. First, the parameter ξ implies a cross-restriction between the skill bias of the adopted technology and the long-run elasticity of substitution between high- and low-skill labor. Second, it features a market-size effects in the process of technology adoption, parameterized by the exponent $(1+\xi)/(\alpha+\xi) > 1$ in the right-hand side of (1.15).

Weaker Market Size Effect

Our model implies a strong market size effect. In this section, we test the robustness of the results to a more general functional form for technology adoption implying that the cost of adopting new technologies may increase in market size. We assume that:

$$c_{LS} = \mu \left(\frac{A_{LS}}{A_{LN}} \right)^{\xi} (L_S)^{\phi} \quad \text{and} \quad c_{HS} = \mu \left(\frac{A_{HS}}{A_{HN}} \right)^{\xi} (ZH_S)^{\phi} ,$$

where $\phi \geq 0$. This model nests the benchmark case in (1.13) when $\phi = 0$. This specification scales up the relative cost of technology adoption by the factor $(ZH_S/L_S)^{\phi}$ compared to the benchmark case and (partly) compensates for the market size effect in relative profits if $\phi > 0$. This allows us to analyze models with a weaker market size effect. Relative output is then given by

$$\frac{Y_S}{Y_N} = \left(\left(\frac{K_S}{K_N} \right)^{\alpha} \left[\frac{L_S^{\frac{(\epsilon-1)(1+\xi-\phi)}{1+\epsilon\xi}} + \left(Z\tilde{h}_N \right)^{\frac{\xi(\epsilon-1)(\varepsilon-1-\varepsilon\phi)}{1+\epsilon\xi}} \times (ZH_S)^{\frac{(\epsilon-1)(1+\xi-\phi)}{1+\epsilon\xi}}}{L_N^{\frac{(\epsilon-1)(1+\xi-\phi)}{1+\epsilon\xi}} + \left(Z\tilde{h}_N \right)^{\frac{\xi(\epsilon-1)(\varepsilon-1-\varepsilon\phi)}{1+\epsilon\xi}} \times (ZH_N)^{\frac{(\epsilon-1)(1+\xi-\phi)}{1+\epsilon\xi}}} \right]^{\frac{(1-\alpha)(1+\epsilon\xi)}{(\epsilon-1)(1+\xi-\phi)}} \right)^{\frac{1+\xi-\phi}{\alpha+\xi}} ,$$

which shows that the long-run elasticity of substitution as well as the scale effect is affected by the parameter ϕ.

We estimate the model under this alternative specification. The constraint that $\xi \geq 0$ turns out to be binding for sub-Saharan countries. The estimated value of ϕ is 0.584 (s.e. 0.006) for year 1970 and 0.576 (s.e. 0.007) for year 2000. The estimated values of ξ for 1970 are 0.63 (s.e. 0.30) for OECD countries and 0.24 (s.e. 0.10) for non-OECD countries (excluding sub-Saharan countries). The corresponding values for year 2000 are 1.18 (s.e. 0.47) for OECD countries and 0.18 (s.e. 0.08) for non-OECD countries (excluding Sub-saharan countries). The goodness of fit, $\Re^2 = 0.934$ in 1970 and $\Re^2 = 0.926$ in 2000, is marginally higher than in the benchmark model.

The results are qualitatively consistent with those in the benchmark model: the estimate of ξ increases significantly (by a factor of two) between 1970 and 2000 for OECD countries, while there is no significant change (the point estimate being in fact somewhat lower) for non-OECD countries. However, the estimated barriers are significantly larger for all countries, or equivalently the elasticity of technology adoption to the distance to the frontier is lower. In addition, it appears as if there is no technology spillover to sub-Saharan countries that develop their technologies in complete isolation ($\xi = 0$). It is worth remarking that the improvement in the fitness is only marginal, indicating that the data cannot discriminate clearly between the two models.

Openness

In our analysis, we have followed the tradition of the development accounting literature assuming all economies to be closed. Free trade was only considered as a counterfactual. However, the absence of trade is a straightjacket, especially for more recent years. For this reason, in this section we re-estimate the model under the alternative assumption that there is free trade in year 2000, based on the results of Proposition 2.

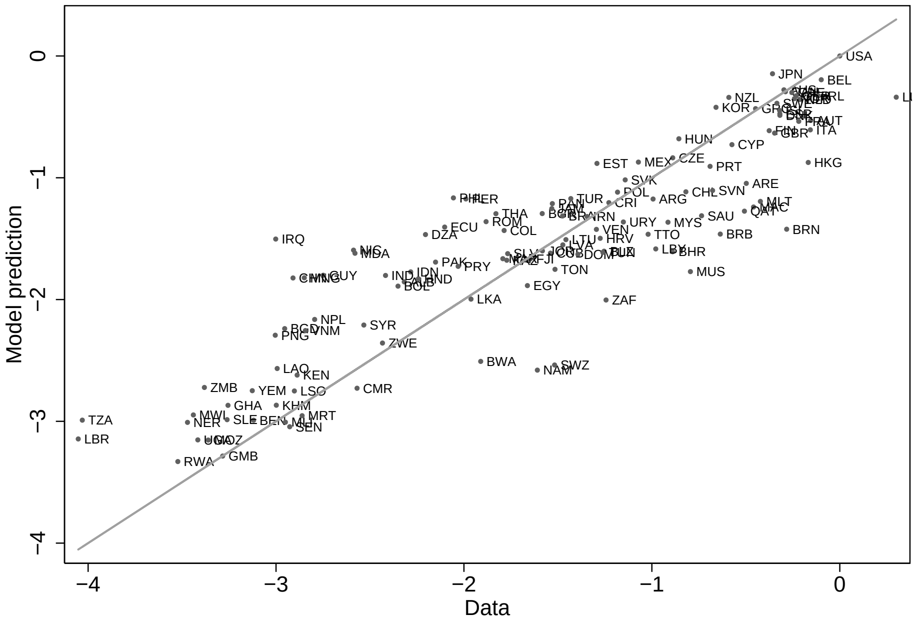


Figure 1.8: Open economy estimation: GDP pw (log-difference from the US).

Under free trade, the estimated barriers for OECD countries become very small, i.e., the estimated ξ is very high and also imprecisely estimated. The restriction that there are no technological barriers for OECD countries cannot be rejected at standard confidence levels.[14] Therefore, we impose the constraint that $\xi \to \infty$ for OECD countries. We report the result of the estimation using tertiary schooling from BL as the measure of skill. This results in $\xi = 10.06$ (s.e. 1.81) for the non-OECD non-sub-Saharan countries and $\xi = 3.88$ (s.e. 0.63) for the sub-Saharan countries. Therefore, estimating the model under free trade yields significantly lower barriers to technology. Interestingly, the open economy model fits better the data ($\Re^2 = 0.934$) than the closed-economy model. Figure 1.8 shows that there is a significant improvement in the fit of emerging economies such as China, India, Indonesia, Thailand, Mexico and Brazil. This is consistent with the observation that these economies are very open to international trade.

[14]In practice, we test that $\xi_{2000} = 1'000'000$ cannot be rejected for the OECD countries.

1.5 Conclusions

In this paper, we have built and estimated a model of the world income distribution based on the following ingredients: different types of labor (skilled and unskilled workers), cross-country differences in factor endowments and in the cost of capital, factor-biased (directed) technical progress and costly technology adoption. Our framework accounts for three sources of income differences: barriers to technology adoption, the inappropriateness (excessive skill-bias) of frontier technologies to local conditions and capital market imperfections. While each of these elements is not new, our contribution is to combine them into a unified empirical model which can be used to gauge the relative importance of different factors generating low productivity and to perform counterfactual experiments.

We summarize here the major findings. First, despite the parsimonious specification, the model provides a good fit of the world income distribution. This suggests that the theory of directed technical change is broadly consistent with aggregate data once properly extended to consider technology adoption and international spillovers. Second, both barriers to adoption and the excessive skill-bias of frontier technologies appear to be quantitatively important. We find that barriers are higher in less developed countries and that they have fallen over time for OECD countries only. The complete removal of barriers would increase output per worker relative to the US (the effect is more pronounced for non-OECD countries) and would lead to higher skill premia. Third, we have used the model to study how the forces of globalization can shape the world income distribution. In the absence of global IPR protection, we find that integration of good markets is followed by SBTC, higher income disparities, and rising skill premia in the majority of countries. These results are however reverted if trade liberalization is coupled with international protection of IPR.

The analysis in this paper can be extended in a number of interesting directions. For instance, we have estimated our benchmark model under the assumption of no international trade and we have then studied globalization as a counterfactual experiment. While this is useful to understand the effects of economic integration, an alternative route would have been to estimate the model taking into account the degree of openness of each country. Finally, although our theory suggests that the removal of barriers to technology adoption has strong distributional consequences, we have not explored how these may generate a political support for the existence of barriers. We believe that including these consideration into the model may shed some light on the important question of which political institutions and reforms can be useful to speed up the much needed process of technological convergence.

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Appendix A

Structural Development Accounting

In this appendix we provide the estimation results for a different sample selection than the one in Table 1.2 of Section 1.4.2. We also provide the analog of Figure 1.2 when ξ is restricted to be the same across countries.

Table A.1: Baseline estimation: constant set of countries

		All countries		OECD		Non-OECD					
						All		Sub-Sahara		Others	
		1970	2000	1970	2000	1970	2000	1970	2000	1970	2000
Data	Skill	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
BL	sec	5.00	4.89	10.72	54.97	4.64	4.20	3.15	3.44	6.10	4.78
		(0.58)	(0.59)	(3.60)	(76.11)	(0.55)	(0.48)	(0.38)	(0.47)	(1.11)	(0.79)
CS	sec	3.86	3.98	6.40	13.77	3.63	3.50	2.17	2.82	4.89	3.90
		(0.39)	(0.43)	(1.09)	(3.88)	(0.39)	(0.38)	(0.27)	(0.32)	(0.62)	(0.59)
BL	tert	3.28	3.28	5.91	17.48	3.08	2.87	2.24	2.17	3.84	3.47
		(0.29)	(0.34)	(1.48)	(9.34)	(0.28)	(0.28)	(0.23)	(0.22)	(0.49)	(0.49)
CS	tert	3.30	2.83	5.53	9.31	3.10	2.49	1.91	1.80	4.13	2.99
		(0.30)	(0.27)	(0.96)	(2.02)	(0.31)	(0.24)	(0.21)	(0.15)	(0.46)	(0.40)
Obs. (BL/CS)		78/68		19/17		59/50		18/16		41/34	

In Table A.1 we repeat the analysis restricting the sample to countries for which information is available both in 1970 and 2000. In 1970, the point estimate for sub-Saharan countries is lower than the point estimate for the other non-OECD countries at the 1 percent level of significance across all specifications. In 2000, it is at least significantly lower at the 5 percent level for the *tert* skill category. For the *sec* skill category, the differences are very close to the 10 percent level of significance. OECD countries have significantly lower barriers than non-OECD countries at the 1 percent level in 2000 for CS (for the BL data we loose the significance), while they are lower at the 5 percent level of significance in 1970 across all specifications. The fit of this model is reported in Table A.2 which is the

Table A.2: Goodness of fit: constant set of countries

		Baseline estimation					
		(1)		(2)		(3)	
Data	Skill	1970	2000	1970	2000	1970	2000
BL	sec	0.918	0.938	0.921	0.948	0.930	0.950
CS	sec	0.928	0.945	0.931	0.953	0.948	0.954
BL	tert	0.896	0.910	0.901	0.927	0.913	0.934
CS	tert	0.917	0.922	0.921	0.936	0.942	0.944

analog of Table 1.3. The predictive power of the model is robust to the considered sample modification, in particular for the specifications in column 2 and 3 where we allow ξ to vary across country groups.

Table A.3: Baseline estimation: BL including Kuwait

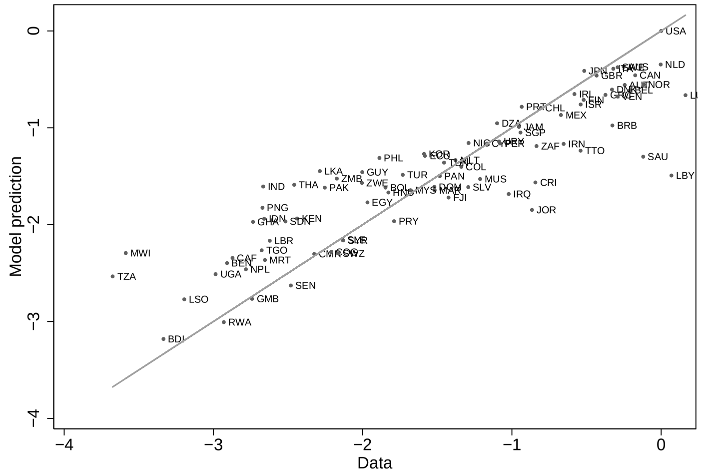
	All countries		OECD		Non-OECD					
					All		Sub-Sahara		Others	
	1970	2000	1970	2000	1970	2000	1970	2000	1970	2000
Skill	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
sec	5.11	5.45	10.72	18.66	4.79	4.88	3.17	3.80	6.90	5.46
	(0.61)	(0.56)	(3.60)	(8.27)	(0.59)	(0.50)	(0.31)	(0.47)	(1.53)	(0.74)
tert	3.32	3.82	5.91	11.71	3.15	3.43	2.25	2.35	4.21	4.12
	(0.30)	(0.34)	(1.48)	(3.37)	(0.29)	(0.31)	(0.19)	(0.24)	(0.63)	(0.48)
Obs.	86	122	19	29	67	93	23	23	44	70

In Table A.3 we present the results of the baseline estimation adding Kuwait to the sample. CS is missing education data for Kuwait, so we restrict the analysis to the BL data set. Kuwait is a strong outlier in terms of GDP pw in 1970, therefore, the point estimate for tertiary schooling in the other non-OECD countries increases from 3.88 (0.47) to 4.21 (0.63) compared to the sample where Kuwait is excluded. It is a general observation that the standard errors go up. However, our main results remain unchanged, only the difference

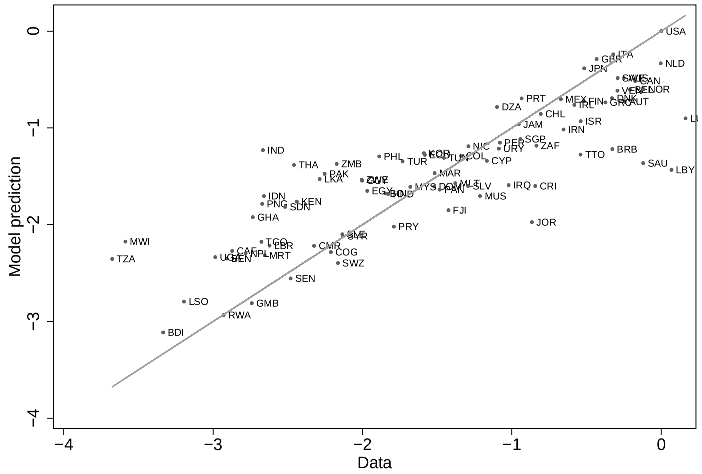
Table A.4: Goodness of fit: BL including Kuwait

	Baseline estimation					
	(1)		(2)		(3)	
Skill	1970	2000	1970	2000	1970	2000
sec	0.914	0.930	0.917	0.935	0.928	0.937
tert	0.891	0.902	0.895	0.911	0.911	0.920

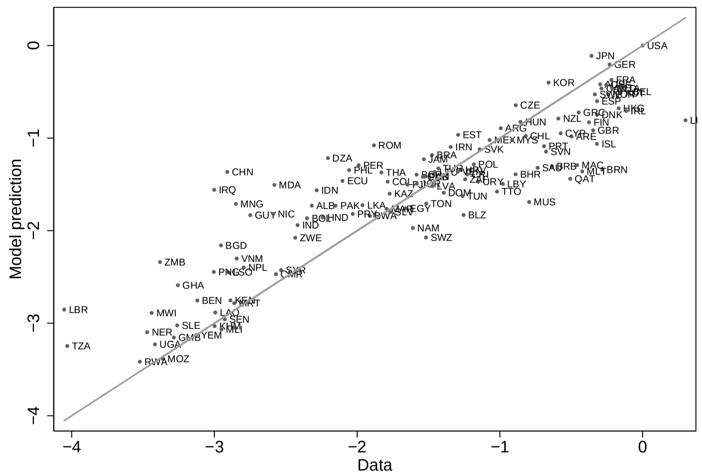
of barriers between OECD and non-OECD countries in 1970 for the secondary schooling



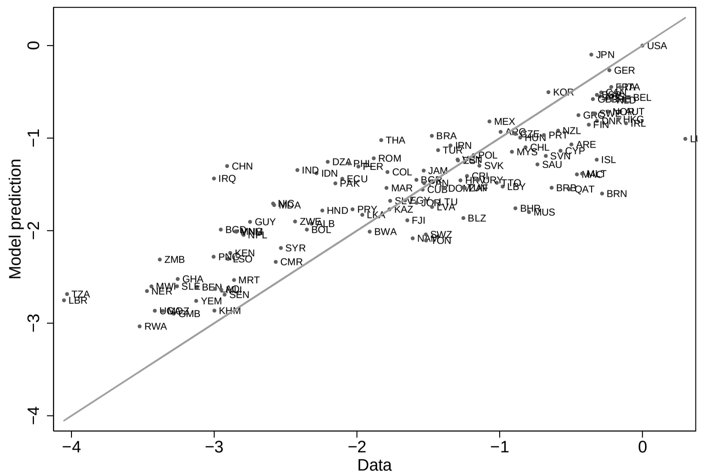
(a) 1970, secondary education



(b) 1970, tertiary education



(c) 2000, secondary education



(d) 2000, tertiary education

Figure A.1: Baseline estimation: GDP pw (log-difference from the US), same ξ across countries.

category falls short of the earlier significance level (10 percent instead of 5 percent). The $\Re^2$s stay high and are reported in Table A.4.

Finally, in Figure A.1 we provide the relative GDP prediction of the baseline model from Section 1.4.2 when we require the same ξ for all countries instead of letting it vary across OECD, sub-Saharan and other non-OECD countries as seen in Figure 1.2. Panels (a)-(d) clearly illustrate that we underpredict income differences for rich countries and underestimate them for poor when imposing a single ξ for all countries. This observation motivates the introduction of income groups in the baseline estimation.

Chapter 2

Optimal Taxation of Foreign Assets and Production Factors in A Small Open Economy

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2.1 Introduction

Recently, proposals for more effective taxation of foreign asset income have returned to the political agenda. A number of proposals have their roots in bilateral, or possibly, multilateral tax agreements. For example, the OECD records a peak in the number of newly signed bilateral Tax Information Exchange Agreements (TIEA) after the beginning of the recent financial crisis. The stock of only 47 TIEAs by the end of 2008 has increased by a flow of 197 new agreements in 2009, 200 in 2010, and 67 in 2011.[1] At the same time, in the course of the ongoing European debt crises, Austria and the United Kingdom made progress in enforcing the taxation of their citizen's foreign asset income from Swiss banks by signing bilateral withholding tax agreements with Switzerland.[2]

While the taxation of foreign asset income is heatedly debated in politics since budgets are tight, the macroeconomic effects of doing so are hardly studied in a dynamic open economy context. The normative dynamic taxation literature[3] with physical capital accumulation, initiated by Judd (1985) and Chamley (1986), has primarily focused on the closed economy. Razin and Sadka (1991b), Correia (1996a) and Atkeson, Chari, and Kehoe (1999) have extended the dynamic analysis to the small open economy, but abstracted from the feasibility of bilateral tax agreements on the taxation of foreign asset income, as well as from fiscal shocks. This paper aims to shed light into this gap of the Ramsey taxation literature and shows how foreign assets and local production factors should be optimally taxed in a small open economy with fiscal shocks and international mobility of capital.

The small open economy setup implies that taxing individual asset income on the *residence principle* is not feasible for the government, because household's can simply avoid such a personal income tax by accumulating assets in a foreign financial institution with a bank secrecy. However, having as an instrument the withholding tax agreement with the abroad economy, the total foreign asset income can be taxed in a nondiscriminatory way. Moreover, also in an open economy the government can observe the input factors of local production which will be physical capital and raw labor in this paper. Thus, the government can tax domestic physical capital rents and and labor wages according to the

[1] http://www.oecd.org/ctp/exchangeofinformation/taxinformationexchangeagreementstieas.htm.

[2] These withholding tax agreements allow the country of residence to tax the *total* foreign asset income, but do not provide access to information on the *individual* accounts of its citizens. Thus, the agreements are in accordance with the Swiss bank secrecy.

[3] With reference to the seminal contribution of Ramsey (1927), this branch of the optimal taxation literature is often labeled Ramsey taxation. A typical setup of the so called Ramsey problem is a benevolent government that commits in the initial period to a sequence of distortionary taxes to finance an exogenous stream of government expenditures. In contrast to the Mirrleesian approach to optimal taxation (Diamond and Mirrlees, 1971), the set of tax instruments available to the government is an additional restriction entering the government's policy choice. The optimal policy that solves the Ramsey problem is called the Ramsey plan, or the Ramsey policy.

source principle.

Taking into account the tax on foreign asset income as an additional policy instrument the paper then derives its main results. First, I show analytically that the optimal policy implements the same allocation as a (hypothetical) residence-based asset income tax where the government's marginal tax revenue from foreign and home asset income is identical. In particular, it is not optimal for the government to set an excessive tax on foreign asset income because such a policy would distort the domestic ratio of physical capital to labor. Second, in the numerical analysis I show that governments should optimally set the tax on foreign asset income in a state-contingent manner to insure itself against the fiscal shocks. This insurance argument has also been made by Chari, Christiano, and Kehoe (1994) for the closed economy: if a good fiscal shock is realized, the government offers citizens a low tax rate - or possibly a subsidy - on foreign asset income, while it sets a high tax rate in fiscal distress. Third, the welfare gains of implementing the optimal tax on foreign asset income can be substantial. Depending on the parametrization of the economy these are measured between 1.7 percent and 0.5 percent of annual consumption in the numerical analysis. However, most of the welfare gains are realized in the transition to the new stationary equilibrium after the introduction of the tax on foreign assets. The value of the new instrument's insurance property is rather limited.

To derive the presented results the paper bases on three main assumptions: (i) the government fully commits to policies announced in the initial period, (ii) the considered open economy and therefore its government is small compared to the rest of the world, (iii) the government balances the budget in every period and cannot run up debt or assets. The above assumption will be discussed in the following along with two minor technical assumptions that concern the stationarity of the model as well as its timing.

While there is a consensus in the literature that no government has direct access to a commitment technology, several mechanisms have been proposed that can replicate it. In particular, Chari and Kehoe (1990) have studied reputational mechanisms to substitute for such a commitment technology. In other contexts, Lucas and Stokey (1983) have used the maturity structure of public debt to mimic optimal fiscal policy with commitment. Persson, Persson, and Svensson (1987) focus on nominal debt to make monetary policy time-consistent. In this paper, I follow the reduced form approach of the dynamic Ramsey taxation literature assuming that the government has access to a perfect commitment technology. This will also allow me to remain comparable with the existing literature. In particular, I model the full commitment to future government policies with a recursive contract between the government and the private agents along the lines of Marcet and Marimon (2011). This approach allows me to characterize the optimal policies in a recursive form and to use standard numerical techniques to solve for the Ramsey plan.

An often considered alternative would be to deviate from the commitment assumption and consider a government that has no access to any form of commitment and mechanisms that replicate it are inoperative. In such a setting, the government can condition its policies only on fundamental states. Krusell, Quadrini, and Ríos-Rull (1996, 1997) and Krusell and Ríos-Rull (1999) were among the first to study such *Markov-perfect* policies in the context of optimal dynamic taxation. The formal definition of the Markov-perfect equilibrium is given in Maskin and Tirole (2001), and Klein, Krusell, and Ríos-Rull (2008) provide a general framework in the context of public policy. Debortoli and Nunes (2010) consider policies under loose commitment where the optimal plans can be periodically revised by future governments with some probability. For the closed economy, it is well documented that in a Markov-perfect equilibrium asset income taxes will be much higher on average than in a Ramsey equilibrium (see Klein and Ríos-Rull (2003), for example).[4] However, the insurance motive of the asset income taxes would be preserved in such a setup. On the other hand, allowing for Markov-perfect policies would affect the welfare analysis provided in this paper, because the value of committing not to tax asset income in future periods would be lost compared to the Ramsey policy presented here.

A small open economy is considered because the focus of this study is on a withholding tax instrument that is supported with bilateral tax agreements between the small open economy and the remaining countries. Such a setup excludes strategic motives in choosing the optimal policy which would result in tax competition between countries. Two-country models of dynamic tax competition with international mobility of capital have been studied in Klein, Quadrini, and Ríos-Rull (2005) and Luthi (2009). The former paper provides a positive theory that aims to match the United States' heavy reliance on capital income taxation compared to Europe. While Klein, Quadrini, and Ríos-Rull (2005) focus on Markov-perfect policies, Luthi (2009) follows the Ramsey tradition assuming full commitment to government policies. In her paper she quantifies the welfare cost of tax competition in comparison with a fiscal union. However, with a balanced budget constraint, the long-run results for the two regimes coincide. So, while I think allowing for tax competition is an interesting extension, I expect that the steady-state analysis would not fundamentally change, but only affect the transitions.

I abstract from the accumulation of government debt because it considerably streamlines the analysis. Moreover, I want to concentrate on the question how governments should trade off different tax instruments in times of tight budgets. Allowing for government debt requires an extensive analysis on its own and is beyond the scope of this paper. The Ramsey taxation literature for the closed economy has found that allowing for state-contingent

[4] In Appendix B.2, I characterize analytically the deterministic long-run foreign asset income tax of the Markov-perfect policy in the context of the presented model.

government debt leads to an indeterminacy[5] of optimal asset income income taxes (Zhu (1992), Chari, Christiano, and Kehoe (1994)). With state in-contingent debt it can be optimal for the government to build up a buffer stock of assets (Aiyagari et al., 2002), a prediction that can not be observed in reality. In the context of dynamic Ramsey taxation, balanced budget rules have first been studied in Stockman (2001) for the closed economy. Klein and Ríos-Rull (2003) have extended the analysis to time-consistent Markov-perfect policies.

A technical complication of the analysis is that exposing the standard small open economy model to a stochastic environment may lead to non-stationary consumption and asset levels. Schmitt-Grohé and Uribe (2003) compare several proposals that have been brought up in the literature to close the stochastic small open economy model. They conclude that all the considered approaches lead to very similar business cycle dynamics. In the environment of this analysis it will be convenient to follow their approach of assuming an increasing portfolio cost from deviating from a long-run level of assets for households. These cost can be interpreted as a measure of the long-run price elasticity of assets, which, in the numerical analysis, I will parameterize to a range of small values including the case where these cost are negligible and assets are perfectly elastic.

Finally, a trivial solution to most Ramsey problems with asset accumulation is that the government optimally taxes initial period assets lump-sum up to the present-value of future government expenditures. This policy is optimal because it avoids distortionary taxation in all other periods. To make the problem interesting, and arguably also more realistic, the literature restricts the Ramsey problem in two different ways. Correia (1996a) and Luthi (2009), for example, restrict the tax rate on asset income to be smaller than some upper bound. The result is that the non-depreciated assets of the initial period is simply taxed at the maximum rate in the following periods and declines thereafter. Chari, Christiano and Kehoe (1994), on the other hand, propose that the government inherits the initial tax on asset income from the past which rules out excessive initial period taxation with a simple timing assumption. I follow the latter approach to avoid an arbitrary upper bound on excessive taxation in initial periods.

Closely related to my work is the paper of Klein and Ríos-Rull (2003) who study the optimal dynamic taxation of labor and asset income in a stochastic but closed economy with a balanced budget constraint. However, these authors focus on the differences between the government's optimal Ramsey plan and the Markov-perfect policy, while I concentrate on Ramsey taxation in an open economy with imperfect enforceability of personal income

[5]Intuitively, the same insurance that the government provides with asset income taxes, could be provided with state-contingent debt: in case of fiscal distress the government offers debt holders a low interest rate, while it sets a high interest when a good fiscal shock is realized.

taxation. For the Ramsey plan, Klein and Ríos-Rull (2003) find that the tax burden of government expenditures is borne almost completely by labor income whereas asset income taxation is used to insure against fiscal expenditure surprises. Because the optimal tax scheme in my model replicates properties of a residence-based asset income taxation scheme, their qualitative results for the closed economy have an analog in the small open economy if the government has at hand the tax on foreign asset income as a policy instrument. But the quantitative results differ, because in the open economy risk can not only be diversified across time and states, but also across countries.

In a broader sense, this paper is also related to the literature on optimal capital controls. In a two-period context, Razin and Sadka (1991a) study the effects of capital controls in the presence of capital flight, and Jeanne and Korinek (2010) show that restricting capital inflows in boom times increase welfare in the presence financial market imperfections. Costinot, Lorenzoni, and Werning (2012) study in a two-country model how governments can use a tax on foreign assets to strategically manipulate the dynamic terms-of-trade. And Jeanne (2011) studies how capital controls can be used to permanently undervalue the real exchange rate. Finally, in the international trade context, the paper also relates to the international taxation literature reviewed in Dixit (1985) and Gordon and Hines (2002).

The remainder of this paper is organized as follows. Section 2.2 presents the benchmark environment of the small open economy without the tax on foreign assets. The associated Ramsey problem and the optimal policies are characterized in Section 2.3. In Section 2.4, the tax instrument on foreign assets is introduced. The numerical simulation of the stochastic steady-state moments and the welfare analysis is provided in Section 2.5, and Section 3.6 concludes.

2.2 Benchmark Environment

This section establishes a benchmark small open economy where the government can only observe and tax factors of local production, namely physical capital and raw labor. This is an economy with a government that lacks an international withholding tax agreement on foreign asset income such that resident household's unobservable assets cannot be taxed. In Section 2.4, I will study the macroeconomic effects of introducing such a bilateral tax agreement and compare it to the benchmark economy laid out here.

The considered stochastic small open economy model is populated by households, firms and a benevolent government. Firms produce a homogeneous good which is internationally tradable and can be used for private and public consumption as well as savings. Private agents have access to an international financial market which transforms household's assets into physical capital at no cost. Households are not allowed to change their residence, so labor is only domestic. In this benchmark economy, the government taxes labor income of households and physical capital rents paid by firms at rates $\tau^n(s^t)$ and $\tau^k(s^t)$, respectively. Uncertainty is modeled in a standard way. In each period t, an exogenous and stochastic state s_t is realized. The state s_t is driven by a Markov process with finite support. Finally, let $s^t = (s_0, \dots, s_t)$ denote the history of realized states up to period t.

2.2.1 Households

Aggregate household behavior can be described by a representative household choosing state-contingent consumption, $c(s^t)$, leisure time, $\ell(s^t)$, and the level of assets, $a(s^t)$, for each $t \in \{0, 1, \dots\}$ to

$$\max \sum_{t,s^t} \mathrm{E}_{s^t|s_0} \beta^t u\left(c(s^t), \ell(s^t)\right),$$

subject to the period-by-period private budget constraint

$$\begin{aligned} & c(s^t) + a(s^t) + q\big(a(s^{t-1})\big) \\ & \quad \leq \big(1 - \tau^n(s^t)\big)\big(L - \ell(s^t)\big)w(s^t) + a(s^{t-1})\left[1 + r(s^t) - \delta\right], \end{aligned}$$

with the initial states s_0 and a_0 given. The price of the final good is normalized to one and the asset $a(s^t)$ represents a deposit in the international financial market yielding the return $r(s^t) - \delta$, where $r(s^t)$ is the domestic rental rate and δ is the depreciation rate of physical capital. Asset holdings of household's are assumed to be subject to the borrowing constraint[6]

$$a(s^t) \geq 0. \tag{2.1}$$

Finally, to close the stochastic small open economy model, I assume that household faces a convex portfolio cost, $q(a)$, along the lines of Schmitt-Grohé and Uribe (2003).[7]

[6] For a formulation without such a borrowing constraint but with transversality conditions see Stockman (2001).

[7] As proposed in Schmitt-Grohé and Uribe (2003), the cost function, $q(a) \equiv q(a - \bar{a})$ is such that deviations from an exogenous long-run asset level $\bar{a}$ are punished at an increasing rate. This imposes stationarity on the asset level and therefore on consumption. The convexity of this portfolio cost is proportional to the long-run price elasticity of assets and in the numerical analysis I will parameterize this convexity to a range of small values including the case where these cost are negligible.

2.2.2 Production

The production sector can be summarized by a representative firm who produces the final good, $y(s^t)$, using physical capital, $k(s^t)$, and raw labor, $n(s^t)$, with the constant returns to scale technology

$$y(s^t) = f(k(s^t), n(s^t)).$$

The production sector is competitive and the final good is the same across countries. The government levies a tax $\tau^k(s^t)$ on physical capital rents paid by local firms to the international financial market. Profit maximization then implies that the tax-adjusted rental rate of physical capital, $r(s^t)(1 + \tau^k(s^t))$, and the domestic wage, $w(s^t)$, are given by their marginal product

$$\begin{aligned} r(s^t)\big(1 + \tau^k(s^t)\big) &= f_1\left(k(s^t), n(s^t)\right) \\ w(s^t) &= f_2\left(k(s^t), n(s^t)\right). \end{aligned}$$

2.2.3 Government

The government chooses the state-contingent tax instruments on the local production factors, $\tau^n(s^t)$ and $\tau^k(s^t)$, to finance exogenous government expenditures, $g(s^t)$, in every state of the history s^t, such that

$$g(s^t) \leq \tau^n(s^t)(L - \ell(s^t))w(s^t) + \tau^k(s^t)k(s^t)r(s^t),$$

where government expenditures are nonproductive[8] and driven by the exogenous stochastic Markov process

$$\Gamma\big(g(s^{t+1})|g(s^t)\big). \tag{2.2}$$

As in Stockman (2001) and Klein and Ríos-Rull (2003), I assume that the government cannot issue debt or accumulate assets. This allows me to focus on a state where the government has to insure against fiscal surprises by trading off the available tax instruments.

[8]This paper focuses on the optimal tax policy in response to expenditure shocks and it is not crucial whether these shocks are modeled as unexpected deviations from an exogenous or an endogenous level of public expenditures. Moreover, as long as public consumption is separable from private consumption in household's preferences, also the long-run optimal level of taxes is unaffected. However, the long-run analysis would change if government expenditures enter the production function as an untaxed factor (Correia, 1996a), for example.

2.2.4 Markets

Because agents cannot change their country of residence, the labor market is only domestic. Domestic market clearing requires

$$n(s^t) = L - \ell(s^t), \tag{2.3}$$

where L denotes the total time endowment of households. The small open economy can run a non-balanced capital and trade account, but is subject to international price equalization

$$r(s^t) = r^\star,$$

where $r^\star$ denotes the international rental rate of physical capital in the international financial market.

2.2.5 Decentralized Equilibrium

Given the sequence of state-contingent tax policies

$$\left\{\tau^k(s^t), \tau^n(s^t)\right\}_{t,s^t},$$

and a world rental rate, $r^\star$, a decentralized equilibrium is a state-contingent sequence of choices

$$\left\{c(s^t), \ell(s^t), k(s^t), a(s^t)\right\}_{t,s^t},$$

which is consistent with the borrowing constraint on assets (2.1), rational expectations about the transition of government expenditures (2.2) and tax policy, labor market clearing (2.3), the intratemporal labor-leisure trade-off,

$$0 = u_2\left(c(s^t), \ell(s^t)\right) - u_1\left(c(s^t), \ell(s^t)\right)\left(1 - \tau^n(s^t)\right) f_2\left(k(s^t), n(s^t)\right), \tag{2.4}$$

the intertemporal Euler equation

$$\begin{aligned} 0 = \Big[& \beta \mathrm{E}_{s^{t+1}|s^t} u_1\big(c(s^{t+1}), \ell(s^{t+1})\big)\Big[1 + r^\star - \delta - q_1\big(a(s^t)\big)\Big] \\ & - u_1\left(c(s^t), \ell(s^t)\right)\Big] a(s^t), \end{aligned} \tag{2.5}$$

the international no arbitrage condition

$$0 = \frac{f_1\left(k(s^t), n(s^t)\right)}{1+\tau^k(s^t)} - r^\star, \tag{2.6}$$

and the private budget constraint

$$\begin{aligned} 0 = &\left(1-\tau^n(s^t)\right)\left(L-\ell(s^t)\right) f_2\left(k(s^t), n(s^t)\right) + a(s^{t-1})\left(1+r^\star-\delta\right) \\ &- \left[c(s^t) + a(s^t) + q\left(a(s^{t-1})\right)\right], \end{aligned} \tag{2.7}$$

where the initial conditions s_0 and a_0 are taken as given and functions with numbered subscripts denote partial derivatives with respect to the corresponding argument. The formulation of the decentralized equilibrium for this benchmark environment shows that the government's fiscal instruments will distort the intratemporal allocation of consumption and leisure, and possibly the international allocation of capital. However, the intertemporal saving decision of households in Equation (2.5) is not affected by the available policy instruments. This will no longer be the true when the government also has at hand the tax on foreign assets income.

2.3 Ramsey Problem

The benevolent government of the small open economy chooses its tax policy in period zero to maximize the equilibrium welfare of domestic agents subject to the government budget constraint and the decentralized behavior of agents. More formally, it chooses allocations, $c(s^t)$, $\ell(s^t)$, $k(s^t)$, $a(s^t)$, and tax policy $\tau^k(s^t)$, $\tau^n(s^t)$, to

$$\max \sum_{t,s^t} \mathrm{E}_{s^t|s_0} \beta^t u\left(c(s^t), \ell(s^t)\right),$$

subject to Equations (2.1)-(2.7) and the government budget constraint

$$g(s^t) \leq \tau^n(s^t)\left(L-\ell(s^t)\right) f_2\left(k(s^t), L-\ell(s^t)\right) + \tau^k(s^t) k(s^t) r^\star, \tag{2.8}$$

given s_0 and a_0. A common feature of dynamic Ramsey problems is that the one-period forward looking constraint in the form of the private Euler equation (2.5) may lead to a time-inconsistent Ramsey plan of the form described in Kydland and Prescott (1977). Namely, in period zero the government might announce a Ramsey plan that it would want to reoptimize at later stages. As is common in the dynamic Ramsey taxation literature, I will assume that the government can fully commit to policies announced in the initial period.[9] However, the time-inconsistency in the Ramsey plan remains a major technical challenge because with forward-looking constraints standard dynamic programming cannot be applied.[10] I will address this issue by modeling the commitment technology as a recursive contract between the government and the private agents along the lines of Marcet and Marimon (2011).

2.3.1 Primal Formulation

I follow the primal approach to Ramsey taxation as proposed in Lucas and Stokey (1983) and applied to economies with balanced-budget constraints in Stockman (2001) to substitute out the tax rates from the constraints of the Ramsey problem. This approach allows me to solve for the optimal allocation directly, and the optimal linear tax system then follows from calculating the tax rates given the optimal allocation. In particular, the open economy model under consideration has two non-redundant tax instruments that allow me to substitute out two constraints of the Ramsey problem. First, the efficiency condition (2.4) can be used to reduce the labor income tax, $\tau^n(s^t)$, in the equilibrium private budget constraint (2.7) to yield

$$\begin{aligned} 0 = u_2\big(c(s^t), \ell(s^t)\big)\big(L - \ell(s^t)\big) + u_1\big(c(s^t), \ell(s^t)\big)\Big[a(s^{t-1})\big(1 + r^\star - \delta\big) \\ - \big[c(s^t) + a(s^t) + q\big(a(s^{t-1})\big)\big]\Big]. \end{aligned} \tag{2.9}$$

Second, adding private and public budgets from Equations (2.7) and (2.8), respectively, using the constant returns to scale property of the domestic production technology, and substituting out the tax on capital rents, $\tau^k(s^t)$, with the international no arbitrage condition (2.6) results in the domestic resource constraint

$$\begin{aligned} 0 \le f\left(k(s^t), L - \ell(s^t)\right) + \big(a(s^{t-1}) - k(s^t)\big)r^\star \\ - [c(s^t) + a(s^t) - (1-\delta)a(s^{t-1}) + g(s^t) + q\big(a(s^{t-1})\big)]. \end{aligned} \tag{2.10}$$

Compared to the resource constraint in a closed economy, the net factor income from abroad, $(a(s^{t-1}) - k(s^t))\, r^\star$, enters the resource constraint as an additional term. Having reduced labor supply and the two tax instruments with the constraints in (2.3), (2.4), and

[9] See Barro and Gordon (1983), Lucas and Stokey (1983), Persson, Persson, and Svensson (1987), and Chari and Kehoe (1990) for mechanisms that can substitute for such a technology. In Appendix B.2, I also provide analytical results for long-run properties of the optimal Markov-perfect policies where the government cannot commit to announced policies.

[10] Standard dynamic programming can only be applied if the Ramsey plan is a time-*invariant* function of the natural state space (a, g). In the case of time-inconsistency, the Ramsey plan is a time-*variant* function of the natural state space (a, g).

(2.6), as well as turned the government budget constraint into the resource constraint, the primal formulation of the government's decision problem reads

$$\max_{c(s^t),\ell(s^t),k(s^t),a(s^t)} \sum_{t,s^t} \mathrm{E}_{s^t|s_0} \beta^t u\left(c(s^t), \ell(s^t)\right), \tag{2.11}$$

subject to (2.1), (2.2), (2.5), (2.9) and (2.10), given s_0 and a_0.

2.3.2 Recursive Formulation

The primal formulation of the dynamic Ramsey problem in (2.11) can be formulated along the lines of a recursive contract between the government and private agents as proposed by Marcet and Marimon (2011).[11] The recursive form of the associated Lagrangian then reads

$$\begin{aligned} W(a,\mu,g) = \min_{\mu'} \max_{c,\ell,k,a' \geq 0} & \ u(c,\ell) - \mu' u_1(c,\ell)a' + \mu u_1(c,\ell)\Big[1 + r^\star - \delta - q_1(a)\Big]a \\ & + \beta \mathrm{E}_{g'|g} W(a',\mu',g') \end{aligned}$$

subject to the resource constraint

$$0 \leq f(k, L-\ell) + (a-k)r^\star - \big[c + a' - (1-\delta)a + g + q(a)\big],$$

the private budget constraint

$$0 = u_2(c,\ell)(L-\ell) + u_1(c,\ell)\Big[a(1 + r^\star - \delta) - [c + a' + q(a)]\Big],$$

with s_0, $\mu_0 = 0$, a_0, and $\Gamma(g'|g)$ taken as given. In the recursive formulation, the multiplier on the past forward-looking Euler equation, μ, enters as an additional state variable. In the reward function, μ keeps track of the government's past policy promises that have been made through last period's Euler equation, and μ' makes sure that current choices will be a binding promise for the future policies via this period's Euler equation. The introduction of the pseudo state variable, μ, makes the Ramsey problem recursive on the augmented state space, (a,μ,g). Based on this augmented recursive form, standard recursive methods can be applied to characterize the Ramsey plan. Given the optimal Ramsey plan, the optimal

[11] A general treatment of recursive methods for dynamic incentive problems is provided in Messner, Pavoni, and Sleet (2012).

tax scheme then follows from the tax wedges implied by the decentralized equilibrium,

$$\tau^n = 1 - \frac{u_2(c,\ell)}{u_1(c,\ell) f_2(k, L-\ell)}, \tag{2.12}$$

$$\tau^k = \frac{f_1(k, L-\ell)}{r^\star} - 1. \tag{2.13}$$

2.3.3 Optimal Policy

It turns out that the Ramsey plan in the benchmark economy is such that the government uses only the labor income tax to finance government expenditures. Because physical capital, k, only affects the resource constraint of the Ramsey problem's recursive formulation, physical capital is optimally allocated to maximize resources in every period given the remaining variables. The corresponding equilibrium condition reads

$$0 = f_1(k, L-\ell) - r^\star. \tag{2.14}$$

Comparing this allocation with the equilibrium no arbitrage condition (2.13) implies an optimal zero tax on physical capital rents, $\tau^k = 0$, in all periods. The intuition for this result is similar to the one provided in Gordon (1983). Because the supply of physical capital is perfectly elastic in this small open economy, the entire burden of either a tax on physical capital rents or labor income is borne by labor. Therefore, it is efficient to tax labor income directly, because the tax on capital rents induces an additional distortion on the allocation of capital. As a direct consequence of this result, the full burden of fiscal expenditures is borne by labor income which is taxed at the rate,

$$\tau^n = \frac{g}{(L-\ell)\, f_2\,(k, L-\ell)}.$$

Since the labor income tax, τ^n, is fully determined by the government's budget constraint the government could never deviate from the announced tax policy *ex post* even if it were allowed to break the recursive contract signed in period zero. Or, in other words, given $\tau^k = 0$ the labor income tax is set in a purely mechanical way. This implies that the Ramsey plan is time-consistent and the additional state is not needed to make the Ramsey policy recursive, $\mu = 0$. Given the optimal state-contingent tax policies described above, the optimal allocation follows immediately from the decentralized equilibrium stated in Equations (2.1)-(2.6).

2.4 Taxation of Foreign Asset Income

This section introduces the withholding tax on foreign asset income to the benchmark environment presented in Sections 2.2 and 2.3. To avoid the rather implausible and trivial case of lump-sum taxation in the initial period, I follow the timing assumption proposed by Chari, Christiano, and Kehoe (1994) that is also applied in Klein and Ríos-Rull (2003), assuming that initial tax on foreign assets income, τ_0^x, is inherited from the past. For all $t \geq 1$ the tax rates, $\tau^x(s^t)$, can be chosen in a state-contingent manner. The government budget constraint then reads

$$\begin{aligned} g(s^t) \leq \tau^n(s^t)\left(L - \ell(s^t)\right) f_2\left(k(s^t), L - \ell(s^t)\right) + \tau^k(s^t)k(s^t)r(s^t), \\ + \tau^x(s^t)\big(a(s^{t-1}) - k(s^t)\big)r^\star. \end{aligned}$$

Depending on the net foreign asset position, the tax $\tau^x(s^t)$ can also turn into a subsidy if the net foreign asset position is negative. However, as will be later shown in the equilibrium analysis, the total tax revenue from taxing foreign asset income *and* domestic capital rents will always be positive.

2.4.1 International No Arbitrage

The tax on foreign asset income affects the international no arbitrage condition in the international financial market,

$$r^\star = r(s^t) + \tau^x(s^t)r^\star \quad \Leftrightarrow \quad \big(1 - \tau^x(s^t)\big)r^\star = r(s^t). \tag{2.15}$$

To understand this no arbitrage condition think of the international financial market as an internationally operating bank that is subject to the foreign asset income tax collected by the domestic government. Then the bank's return to allocating one unit of physical capital abroad, $r^\star$, has to equal the return to allocating one unit of physical capital in the domestic economy, $r(s^t)$, plus the reduction in the foreign asset income tax, $\tau^x(s^t)r^\star$. As a consequence, the private Euler equation of the decentralized equilibrium becomes

$$\begin{aligned} 0 = \Bigg[&\beta \mathrm{E}_{s^{t+1}|s^t} u_1\big(c(s^{t+1}), \ell(s^{t+1})\big)\Big[1 + \big(1 - \tau^x(s^{t+1})\big)r^\star - \delta - q_1\big(a(s^t)\big)\Big] \\ &- u_1\left(c(s^t), \ell(s^t)\right)\Bigg]a(s^t), \end{aligned} \tag{2.16}$$

and the international no arbitrage condition now reads

$$0 = \frac{f_1\left(k(s^t), n(s^t)\right)}{\left(1+\tau^k(s^t)\right)\left(1-\tau^x(s^t)\right)} - r^\star. \tag{2.17}$$

Thus, a positive value of $\tau^x(s^t)$ has the same effect as would a tax on resident household's individual asset income and a subsidy on physical capital rents.

2.4.2 Ramsey Problem

Because the initial tax on foreign asset income is inherited from the past, the resulting Lagrangian of the Ramsey problem is only recursive on the augmented state space, (a, μ, g) from period one onwards. Since he focus of this section will be on the stochastic long-run properties of the equilibrium, I will abstract from the optimal period zero allocation in the remainder of this section which is not crucial. The recursive form of the augmented Lagrangian from period one onwards reads[12]

$$\begin{aligned} W(a,\mu,g) = \min_{\mu'} \max_{c,\ell,k,a' \geq 0} & \; u(c,\ell) - \mu' u_1(c,\ell)a' \\ & + \mu\Big[u_1(c,\ell)\big[c + a' + q(a) - aq_1(a)\big] - u_2(c,\ell)(L-\ell)\Big] \\ & + \beta \mathrm{E}_{g'|g} W(a',\mu',g') \end{aligned} \tag{2.18}$$

subject to the resource constraint

$$0 \leq f(k, L-\ell) + (a-k)r^\star - [c + a' - (1-\delta)a + g + q(a)],$$

with $a(s_0)$, $\mu(s_0)$, s^1, and $\Gamma(g'|g)$ given. Note that having the tax on foreign asset income as a policy instrument is the same as removing a constraint from the Ramsey problem of the benchmark economy. Thus equilibrium welfare must be weakly higher compared to the benchmark economy. Given the optimal allocation, the optimal linear tax scheme for all $t \geq 1$ follows from equation (2.12), and

$$\begin{aligned} \tau^k &= \frac{f_1(k, L-\ell)}{(1-\tau^x)r^\star} - 1, \\ \tau^x &= 1 - \frac{c + a' - (1-\delta)a + q(a) - (1-\tau^n)(L-\ell)f_2(k,L-\ell)}{ar^\star}. \end{aligned}$$

[12]The derivation of the primal formulation and the associated Lagrangian of the Ramsey problem from period zero onwards is delegated to Appendix B.1. The primal formulation is an application of the proposition stated in Stockman (2001).

In period 0, τ_0^x is inherited from the past, while the formulas for the other tax rates remain unchanged.

2.4.3 Optimal Tax on Physical Capital Rents

The equilibrium condition with respect to the optimal allocation of physical capital remains unchanged compared to the benchmark economy is unchanged and given by Equation (2.14). This is not surprising because the supply of physical capital is still perfectly elastic, thus it would be inefficient to distort the allocation of physical capital. However, since the direct tax on foreign asset income implicitly affects capital rents, the direct tax on physical capital rents should be used to exactly offset the distortion that is created by the tax on foreign asset income,

$$\tau^k = \frac{\tau^x}{1-\tau^x}.$$

The total tax revenue from foreign assets and physical capital,

$$\tau^k kr + \tau^x (a-k) r^\star = \tau^k ar = \tau^x ar^\star,$$

then reveals that total assets, a, no matter whether located at home or abroad, are effectively taxed at the same rate, $\tau^k r$. Or, the optimal policy implements the same allocation as would a residence-based asset income tax where the government's marginal tax revenue from foreign and home asset income is, $\tau^k r$ and $\tau^x r^\star$, respectively.

2.4.4 Optimal Labor Income Tax

The optimality conditions with respect to consumption, c, and leisure, ℓ, can be combined with (2.12) to derive the optimal labor income tax rate as a function of the optimal allocation and the state, μ,

$$\begin{aligned}\tau^n = {} & \frac{[u_{11}(c,\ell)f_2(k,L-\ell) - u_{12}(c,\ell)]\,[\mu' a' - \mu[c + a' + q(a) - a q_1(a)]]}{(1+\mu)u_1(c,\ell)f_2(k,L-\ell)} \\ & + \frac{\mu\,[u_{21}(c,\ell)f_2(k,L-\ell) - u_{22}(c,\ell)]\,(L-\ell)}{(1+\mu)u_1(c,\ell)f_2(k,L-\ell)}.\end{aligned}$$

To gain intuition about the sign of the labor tax, suppose that the cross derivative of the utility function is close to zero,[13] which makes the second addend of this expression positive.

[13] For utility functions additively separable in consumption and leisure the argument applies exactly.

The term

$$\mu' a' - \mu[c + a' + q(a) - aq_1(a)],$$

then determines whether the tax rate on labor income is smaller or bigger than zero. In the stochastic steady-state, μ' and μ will take similar values, moreover the portfolio cost of assets will be small compared to consumption expenditures. This then implies a positive labor income tax in the stochastic steady-state which is in line with the existing literature on state-contingent Ramsey taxation in the closed economy (see Chari, Christiano, and Kehoe (1994), and Klein and Ríos-Rull (2003), for example).

2.4.5 Optimal Tax on Foreign Asset Income

The optimality condition with respect to the future asset level pins down the government's Euler equation,

$$\begin{aligned} 0 = \Big[& \beta \mathrm{E}_{g'|g} \lambda' \left[1 + r^\star - \delta - q_1(a')\right] - \lambda + (\mu - \mu')\, u_1(c, \ell) \\ & - \beta \mathrm{E}_{g'|g} \mu' a' q_{11}(a') u_1(c', \ell') \Big] a'. \end{aligned} \tag{2.19}$$

The comparison with the private Euler equation (2.16) reveals that the convexity of the cost function, $q_{11}(a)$, is an important determinant of the long-run level of the optimal tax on foreign assets. Combining the private Euler equation in (2.16) with the government Euler equation in (2.19), the optimal tax on foreign assets in an interior deterministic steady-state can be written as

$$\tau^x = \frac{\mu}{\lambda r^\star} u_1(c, \ell) a q_{11}(a). \tag{2.20}$$

This implies that the convexity of the portfolio cost is proportional to the deterministic long-run tax rate on foreign asset income. The intuition for this result is related to the one provided in Correia (1996b). According to the private Euler equation (2.5), assets' long-run elasticity with respect to the rental rate, $(1 - \tau^x) r^\star = r$, is given by,

$$\varepsilon_{a,r} \equiv \frac{1/\beta - \left[1 - \delta - q_1(a)\right]}{a q_{11}(a)}.$$

Thus, the convexity of the portfolio cost makes assets a less than perfect elastic tax base, and therefore, a more efficient source of tax revenue in the long-run. However, as the portfolio cost goes to zero, assets become perfectly elastic and the entire burden of either a tax on asset income, physical capital rents, or labor income is borne entirely borne by labor. Because the indirect taxation introduces additional distortions, it is efficient to tax labor income directly which implies that the deterministic steady-state tax on foreign asset income will go to zero. However, as is similarly shown in Zhu (1992), Chari, Christiano, and Kehoe (1994) and Klein and Ríos-Rull (2003) for the asset income tax in the closed economy, the stochastic steady-state tax on foreign asset income will fluctuate around a moderate long-run value. This will be shown in the numerical analysis of Section 2.5.

2.5 Numerical Analysis

This section illustrates the stochastic properties of the two tax regimes described in the Sections 2.2 and 2.4, respectively, with a numerical analysis. Henceforth, I label the benchmark economy *without* the tax on foreign asset income the B-Economy, and the economy *with* the tax on foreign asset income the T-Economy. In a first step, I will simulate the model to find the stationary distribution of Ramsey policy. In particular, I want to answer the question how the government of the T-Economy should optimally implement state-contingent taxes on foreign asset income to insure against government expenditures shocks. Moreover, I will also report the implications for the optimal labor income tax policy. The parameterization of the model along the lines of Klein and Ríos-Rull (2003) will allow for a comparison with their results derived for the closed economy. In a further step, I quantify the value of having the additional tax instrument on foreign asset income compared to the B-Economy and illustrate the transitional dynamics to the stationary equilibrium.

2.5.1 Functional Forms and Parameterization

As proposed in Klein and Ríos-Rull (2003), the utility function is of the constant relative risk aversion (CRRA) type, and the production function of the standard Cobb-Douglas form,

$$u(c, \ell) = \frac{\left(c^{\eta}\ell^{1-\eta}\right)^{1-\sigma} - 1}{1-\sigma}, \quad f(k, n) = Ak^{\alpha}n^{1-\alpha}.$$

The preference parameters η and σ capture the relative taste for leisure and the inverse of the intertemporal elasticity of substitution, respectively. The world's rental rate is

$$r^{\star} = 1/\beta - (1-\delta),$$

which is consistent with a stationary and deterministic world economy without portfolio cost. Following Schmitt-Grohé and Uribe (2003), the portfolio cost of the small open economy are of the convex form

$$q(a) = \frac{\phi}{2}(a - \bar{a})^2,$$

where $\bar{a}$ denotes the exogenous long-run target level of assets. The parameter ϕ controls the magnitude and the convexity of these portfolio cost. The targeted long-run level of assets $\bar{a}$ is calibrated to the deterministic steady-state capital stock for each parameterization of the model, such that the portfolio cost are zero when the long-run capital account is balanced. The free total productivity parameter A is calibrated to normalize the output of the deterministic steady-state of the B-economy with a portfolio cost parameter $\phi = 0.05$ to unity and will remain constant across all parameterizations.
As proposed by Klein and Ríos-Rull (2003, Table 2), government expenditures evolve according to a two-state Markov chain with elements, $g \in \{0.184, 0.216\}$, and a symmetric probability state transition matrix with persistence parameter $\rho = 0.835$. The mean government expenditures are then around 20 percent of output. With the exception of the

Table 2.1: Parameterization

Parameter	A	α	β	δ	η	L	$r^\star$	σ
Value	2.04	0.36	0.97	0.08	0.20	1.00	0.11	1.00

parameter ϕ which I let vary over a range of small values, the remaining parameters are chosen as listed in Table 2.1 and taken from Klein and Ríos-Rull (2003).[14]

2.5.2 B-Economy Results

Starting with the benchmark economy where the tax on foreign assets is not available to the government, Table 2.2 reports the simulated moments of the optimal stationary tax rate on labor income. The numerical analysis reveals that the ergodic set of the stationary equilibrium gets very large for values of the portfolio cost parameter ϕ below 0.005, so I will report the numerical results for values above and including this threshold, $\phi \in \{0.02, 0.01, 0.005\}$. The numerical simulation reported in Table 2.2 shows that the government optimally sets an average steady-state labor income tax rate of around 31 percent to finance government expenditures. Because the labor tax is mainly driven by the government budget constraint, it is not surprising that its correlation with the innovations in the government expenditures is positive and well above 0.9. Finally, since the government

[14] The taste for leisure, η, takes a relatively high value resulting in leisure choices around four fifths of the available time, L. Klein and Ríos-Rull (2003) motivate this by the observation that the total time spent on working out of the working age population is around a fifth of the available time.

Table 2.2: Stationary Ramsey plan, B-Economy

Portfolio cost, ϕ	.02	.01	.005	g
Mean	0.31	0.31	0.31	0.2
Standard deviation	0.019	0.018	0.018	0.016
Correlation with g	0.95	0.94	0.93	1

has no access to foreign asset income taxation or government debt the only way to finance fiscal shocks is to use the labor income tax. Because of the distortion on the labor supply this results in a standard deviation for the simulated labor income tax rate that is slightly above the one of government expenditures.

2.5.3 T-Economy Results

Moving on to the case of foreign asset income taxation, Table 2.3 reports the moments of the stationary tax rates, τ^n and τ^x, for the T-Economy.[15] In this setup, the ergodic set of the stationary equilibrium remains stable for values of the portfolio cost above and including $\phi = 0.0001$. The first row of the Table 2.3 reports that the long-run burden of government expenditures is mostly borne by labor income. Only if the portfolio cost are

Table 2.3: Stationary Ramsey plan, T-Economy

Portfolio cost, ϕ	.02		.01		.005		.0001	
Tax rate	τ^n	τ^x	τ^n	τ^x	τ^n	τ^x	τ^n	τ^x
Mean	0.23	0.12	0.26	0.07	0.28	0.04	0.30	0.00
Standard deviation	0.004	0.047	0.004	0.051	0.004	0.054	0.005	0.060
Correlation with g	-0.16	0.98	-0.20	0.97	-0.24	0.97	-0.28	0.97
Correlation with τ^n	1	-0.38	1	-0.43	1	-0.47	1	-0.48

relatively high a partial share of expenditures is also financed through taxing foreign asset income and physical capital rents. As already indicated by Equation (2.20), the mean tax rate on foreign asset income decreases with the portfolio cost parameter ϕ and eventually reaches a level of approximately zero as the cost are vanishing. In the latter case, the average fiscal burden is almost borne only by household's labor income.

A further observation is that the standard deviation of the tax on foreign asset income is around three times that of government expenditures. This pattern becomes more pronounced as the portfolio cost get smaller. At the same time the foreign asset income tax

[15] Remember that the optimal tax rate on physical capital rents is a monotonic transformation of the foreign asset income tax, $\tau^k = \tau^x/(1-\tau^x)$. For small values of τ^x the two tax rates' stationary properties are approximately the same.

is highly positively correlated to government expenditures, thus, unexpected government expenditures are optimally financed through the taxation of foreign asset income and physical capital rents. In short, this means that the contracyclical pattern of asset income taxes in the closed economy derived in Chari, Christiano, and Kehoe (1994) carries over to the small open economy if the government has at hand the tax on foreign asset income.

On the other hand, the labor income tax remains very smooth with a standard deviation less than ten times small that that of the foreign asset income tax. Also, it is negatively correlated to the tax on foreign asset income as well as innovations in government expenditures. Thus, it is optimal to finance a labor income tax cut with an increases in the tax on foreign asset income if an expenditure shock hits the economy. This pattern is more pronounced the higher the mean labor income tax distorted the long-run labor supply.

In general, these observations are qualitatively consistent with the closed economy results reported in Klein and Ríos-Rull (2003, Table 7). However, one important difference is that, quantitatively, their asset income tax rate's standard deviation is two times higher than the one reported in Table 2.3. One important channel this result is that, in the small open economy, fiscal risk can be better diversified through cross-country capital flows. Related to that, the labor income tax cuts in response to a fiscal shock are less pronounced in the open economy because increases in the labor supply are complemented with physical capital inflows that make labor more productive.

Table 2.4 provides a comparison between the mean levels of stationary variables for the B-Economy as well as the T-Economy. A fist observation is that the government of the

Table 2.4: Stationary equilibrium comparison

Portfolio cost, ϕ	.02		.01		.005	
Type-Economy	B	T	B	T	B	T
Assets, a	3.25	2.93	3.25	2.74	3.25	2.63
Capital, k	3.25	3.59	3.25	3.50	3.25	3.45
Foreign assets, $(a-k)/y$	0.00	-0.60	0.01	-0.70	0.01	-0.78
Labor, n	0.17	0.19	0.17	0.18	0.17	0.18
Output, y	1.00	1.11	1.00	1.08	1.00	1.07
Income, $y+(a-k)r^{\star}$	1.00	1.03	1.00	1.00	1.00	0.97
Consumption, c	0.54	0.59	0.54	0.57	0.54	0.56

T-Economy runs a capital account deficit, because the foreign asset income tax distorts the accumulation of assets downwards while the increase in the labor supply generates physical capital inflows. As a direct consequence of the higher capital and labor supply, the T-Economy also produces more final goods. Finally, the mean stationary income of resident household's in the T-Economy falls below the income of the B-Economy as the labor income tax rate gets higher. This maps also into the asset level and consumption which are both falling with the stationary income.

2.5.4 Welfare Analysis

A general property of the Ramsey plan is that the introduction of the tax on foreign asset income weakly increases welfare viewed from the initial period. The informal reason being that the government is provided with an additional tax instrument that it could simply commit to leave aside if it generated welfare losses. Because Table 2.4 reports that the foreign asset income tax depresses the stationary income if the portfolio cost parameter is small, there must be gains of the tax in the transition to the stationary equilibrium that make up for the long-run losses in income. This is in line with the results of Chari, Christiano, and Kehoe (1994) and Stockman (2001), who find that the gains from switching to time-inconsistent Ramsey policy are mostly realized in initial periods.

To quantify the value of having the tax on foreign asset income in the set of policy instruments, I consider the amount x by which consumption in the B-Economy needs to be scaled up in each state of the history s^t to equate the welfare of the T-Economy,

$$\begin{aligned} U^T(a_0, g_0) = u\big(&c^B(a_0, g_0)(1 + x(a_0, g_0)), \ell^B(a_0, g_0)\big) \\ &+ \beta \mathrm{E}_{g'|g} \widetilde{U}^B\big(a_1^B, g_1\big), \end{aligned}$$

where $\widetilde{U}^B(a, g)$ is defined recursively,

$$\begin{aligned} \widetilde{U}^B\big(a, g\big) = u\big(&c^B(a, g)(1 + x(a_0, g_0)), \ell^B(a, g)\big) \\ &+ \beta \mathrm{E}_{g'|g} \widetilde{U}^B\big((a^B)', g'\big), \end{aligned}$$

and the superscript denotes the Ramsey plan of the T-Economy and the B-Economy, respectively. The same welfare measure has been proposed in Stockman (2001) to compare the welfare losses of the introduction of budget rules. For convenience, I choose the initial assets level, a_0, to be the mean of the stationary asset level in the B-Economy, $\bar{a}^B$. The amount $x(\bar{a}^B, g_0)$ can then be interpreted as the welfare gain of unexpectedly introducing the tax on foreign asset income to an average B-Economy in the stationary equilibrium where the government is allowed to reoptimize tax policies after having learned about the additional tax instrument.

Table 2.5 reports the welfare gains in consumption equivalents, $x(\bar{a}^B, g_0)$, for both a high and a low realization of initial government expenditures, g_0. Not very surprisingly, the welfare effects of introducing the tax on foreign asset income depends positively on the convexity of the portfolio cost which are an inverse measure of the long-run price elasticity

Table 2.5: Welfare gains in consumption equivalents

Portfolio cost, ϕ	.02		.01		.005	
Initial state, g_0	low	high	low	high	low	high
Welfare gain (in %)	1.72	1.49	1.11	0.84	0.74	0.45

of assets. Depending on this elasticity, the reported welfare gains vary between 1.7 percent and 0.5 percent of annual consumption in the B-Economy.

Finally, to study the timing of the welfare gains reported in Table 2.5, I illustrate the transitional dynamics of the Ramsey policies with an unexpected introduction of the tax on foreign asset income to an average B-Economy in the stationary equilibrium. In the period of the policy change, the government is allowed to reoptimize policies but the initial tax on foreign asset income is inherited from the past.[16]

Figure 2.1 illustrates the dynamics of the mean tax on foreign asset income, τ^x, the mean labor income tax, τ^n, the mean momentary utility, $u(c, \ell)$, and the mean total asset level, a, in the transition to the new stationary equilibrium after the policy change. The top left panel shows that the tax on foreign asset income peaks in period one[17] after the policy change at rates around seven to nine percent. The bottom right panel illustrates that, in response to the introduction of the foreign asset income tax, agents gradually adjust their asset holdings downwards to the new stationary asset level. After the peak, the optimal tax on foreign asset income is declining with the asset level until the long-run price elasticity of assets kicks in which determines its stationary level. For high values of the portfolio cost, the stationary level of foreign asset income taxes can even be higher that the short-run peak.

Driven by the pattern of the foreign asset income tax, the bottom left panel shows that the momentary utility also peaks in period one after the policy change and then gradually declines to the stationary level as the tax on foreign assets flattens out. Interestingly, the stationary level of the momentary utility in the T-Economy is very similar to the one in the B-Economy, thus the welfare gains of foreign asset income taxation must be realized in the periods of the transition. Consequently, the value of the insurance that can be provided with the foreign asset income tax is limited.

The top right panel shows that it is optimal to lower labor income taxes in the period of the policy change. This pattern is mainly driven by the fact that the initial tax on foreign asset income is predetermined at the zero level. The cut in labor income taxes induces private agents to work more, leading to a drop in the initial momentary utility because

[16] This policy experiment is equivalent to comparing a B-Economy to a T-Economy starting from the same initial asset and expenditure level, $a_0 = \bar{a}^B$ and g_0, respectively, and setting $\tau_0^x = 0$.

[17] In the initial period, the tax on foreign asset income is fixed at the predetermined zero level.

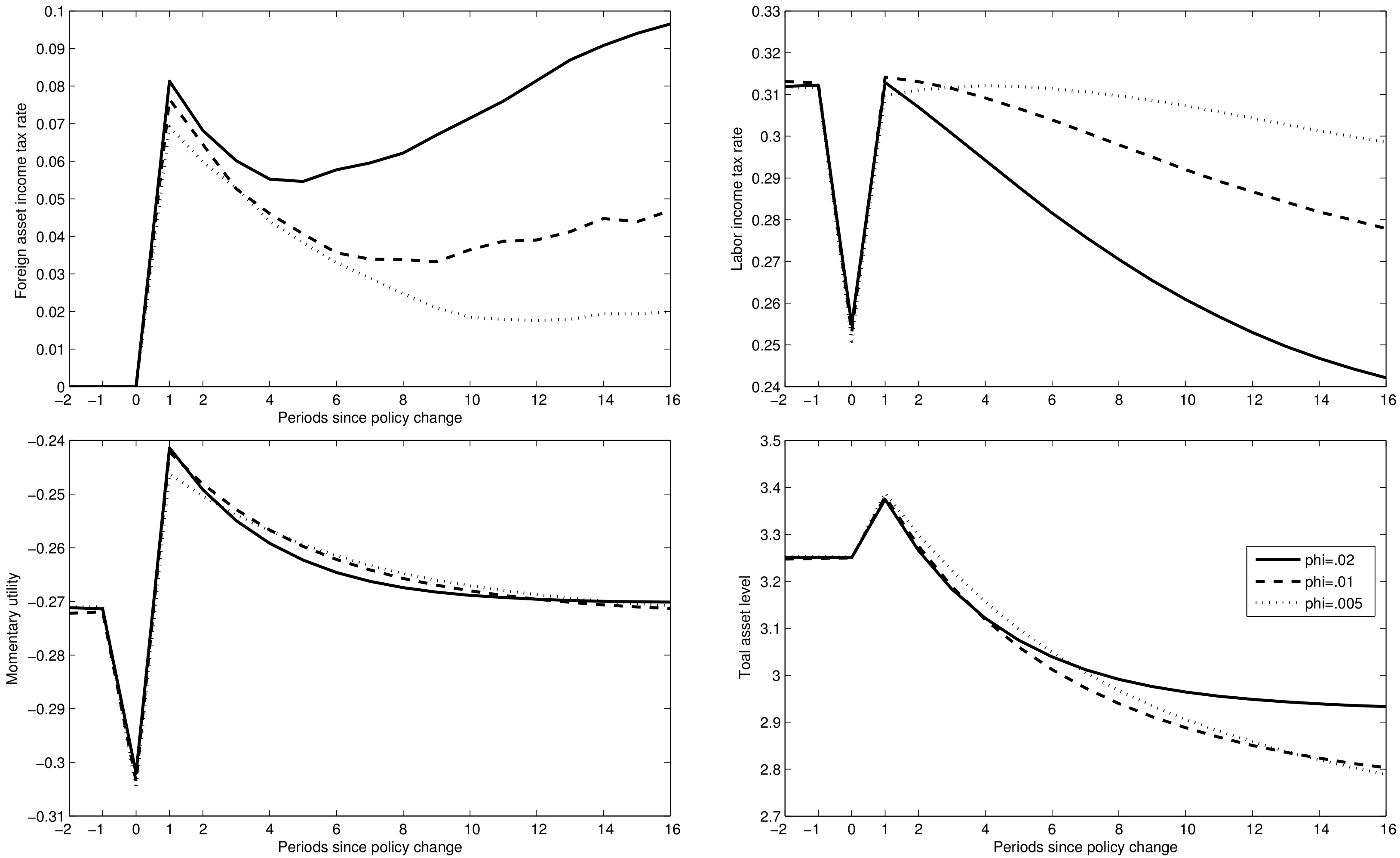

Figure 2.1: Transitional dynamics.

leisure is valued, but also to increased savings which can be taxed from the next period with the foreign asset income tax. This will compensate for the initial drop in momentary utility in later periods. After this period of adjustment, the labor income tax rate gradually adjusts to its stationary level. Inducing a higher stationary level of labor supply the lower the stationary level of the labor income tax.

In summary, the welfare gains of introducing a tax on foreign asset income can be substantial. However, most of the gains are realized in the periods after policy change, and not through the insurance property of the optimal foreign asset income tax.

2.6 Conclusions

In this paper I have studied the dynamic Ramsey taxation of foreign asset income and domestic production factors in a small open economy with international capital mobility, imperfect enforceability of personal asset income taxes and fiscal shocks.

I find that the optimal tax scheme replicates a residence-based asset income tax where the government's marginal tax revenue from foreign and home assets is the same. So, there is no motive for excessive taxation foreign asset income relative to home asset income as has been proposed in the political discussion. Furthermore, the numerical analysis reveals that the government optimally sets the foreign asset income tax in a state-contingent way to insure against fiscal shocks: in times of fiscal distress it puts a high tax on foreign asset income, while in times of low government expenditures it offers a low tax rate. Next to this insurance property, there is no role for foreign asset income taxation in the long-run. The stationary average foreign asset income tax is close to zero, and the expected burden of fiscal expenditures is mostly borne by labor income. While the welfare gain of having the tax on foreign asset income as a policy instrument is substantial, most of those gains are realized in the transition to the stationary equilibrium. Thus, the value of the tax new instrument's insurance property is limited, but the ability to tax partially the existing stock of assets in initial periods is very appealing.

The results of this study can be related to the bilateral tax agreements that have regained momentum in the political discussion of the ongoing European debt crises. Prominent examples are the bilateral withholding tax agreements that Austria and the United Kingdom have signed with Switzerland. In these agreements, Switzerland is asked to collect taxes on asset income of foreign citizens on behalf of their home country. This study suggests that the tax on foreign asset income should be set in a state-contingent way, while at the same time moderate in the expected level if governments can commit to future policies. Moreover, the observation that the tax on foreign asset income is peaking in periods after the introduction of the new tax instrument can be observed in the design of the actual withholding tax agreements between Austria and the United Kingdom with Switzerland.

To complement this study with a more positive analysis, one important direction for future research is the deviation from the assumption that the government can fully commit to announced policies. Contrary to the Ramsey setup, the consideration of Markov-perfect policies, for example, would map the political economy dimension into the model. Reflecting the fact that successive governments tend to reoptimize policies implemented by their predecessors. A technical challenge for the future will be to explore new approaches that allow closing the stochastic small open economy model in an endogenous manner. Another promising direction for future research would be to endogenize the world rental rate such that the asset accumulation across a set of small open economies is consistent with firms demand for physical capital. Such an approach can rule out excessive accumulation of assets.

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Appendix B

Optimal Taxation of Foreign Assets and Production Factors in A Small Open Economy

B.1 Taxation of Foreign Asset Income

This appendix derives the complete primal formulation of the dynamic Ramsey problem with the tax on foreign asset income as described in Section 2.4.

B.1.1 Decentralized Equilibrium

Given the sequence of state-contingent tax policies

$$\left\{\tau^k(s^t), \tau^n(s^t), \tau^x(s^t)\right\},$$

and the world rental rate, $r^\star$, a decentralized equilibrium is a state-contingent sequence of choices

$$\left\{c(s^t), \ell(s^t), k(s^t), a(s^t)\right\}_{t,s^t},$$

consistent with the borrowing constraint on assets (2.1), rational expectations about the transition of government expenditures (2.2) and tax policies, labor market clearing (2.3), the intratemporal labor-leisure trade-off (2.4), the intertemporal Euler equation in (2.16), the international no arbitrage condition (2.17), and the private budget constraint

$$\begin{aligned}
0 = &\big(1-\tau^n(s^t)\big)\left(L-\ell(s^t)\right)f_2\left(k(s^t),n(s^t)\right)\\
&+a(s^{t-1})\left[1+\big(1-\tau^x(s^t)\big)r^\star-\delta\right]-\left[c(s^t)+a(s^t)+q\big(a(s^{t-1})\big)\right], \qquad \text{(B.1)}
\end{aligned}$$

with initial conditions s_0, a_0, and τ_0^x taken as given.

B.1.2 Primal Formulation

The tax on foreign assets can be reduced from the constraint set of the Ramsey problem for all $t \geq 1$ by combining (B.1) and (2.16) to the yield the sequence of implementability constraints,

$$\begin{aligned}
0 = \Bigg[&\beta \mathrm{E}_{s^{t+1}|s^t}\Bigg\{u_1\big(c(s^{t+1}),\ell(s^{t+1})\big)\Big[c(s^{t+1})+a(s^{t+1})+q\big(a(s^t)\big)\\
&-a(s^t)q_1\big(a(s^t)\big)\Big]-u_2\big(c(s^{t+1}),\ell(s^{t+1})\big)\big(L-\ell(s^{t+1})\big)\Bigg\}\\
&-u_1(c(s^t),\ell(s^t))\Bigg]a(s^t), \qquad \text{(B.2)}
\end{aligned}$$

as is similarly shown in the proposition of Stockman (2001). Because the resource constraint (2.10) remains unaffected by the tax on foreign asset income, the primal formulation of the Ramsey problem then reads

$$\max_{c(s^t),\ell(s^t),k(s^t),a(s^t)} \sum_{t,s^t} \mathrm{E}_{s^t|s_0}\beta^t u\left(c(s^t),\ell(s^t)\right)$$

subject to (2.1), (2.2), (2.10), and (B.2), and the private budget constraint from the initial period

$$\begin{aligned}
0 = &\,u_2\big(c(s_0),\ell(s_0)\big)\big(L-\ell(s_0)\big)\\
&+u_1\big(c(s_0),\ell(s_0)\big)\Big[a_0\Big[1+\big(1-\tau_0^x\big)r^\star-\delta\Big]-\big[c(s_0)+a(s_0)+q(a_0)\big]\Big], \qquad \text{(B.3)}
\end{aligned}$$

given s_0, a_0 and τ_0^x. Along the lines of Marcet and Marimon (2011), the associated formulation of the augmented Lagrangian reads

$$\begin{aligned}
W_0\big(a_0,\tau_0^x,g(s_0)\big) = &\min_{\mu(s_0)}\max_{c(s_0),\ell(s_0),k(s_0),a(s_0)} u\big(c(s_0),\ell(s_0)\big)-\mu(s_0)u_1\big(c(s_0),\ell(s_0)\big)a(s_0)\big)\\
&+\beta\mathrm{E}_{s^1|s_0}W\big(a(s_0),\mu(s_0),g(s^1)\big),
\end{aligned}$$

subject to the initial period resource constraint,

$$0 \leq f\left(k(s_0), L - \ell(s_0)\right) + \big(a_0 - k(s_0)\big) r^{\star}$$
$$- \left[c(s_0) + a(s_0) - (1-\delta) a_0 + g(s_0) + q(a_0)\right],$$

and the initial period private budget constraint (B.3). The initial values s_0, a_0, τ_0^x, as well as $\Gamma(g(s^{t+1})|g(s^t))$ are taken as given and $W\left(a(s_0), \mu(s_0), g(s^1)\right)$ is defined recursively in Equation (2.18). Note that this problem is not recursive from period zero, because the period zero allocation is subject to the initial period constraint (B.3) of the private budget imposed by the predetermined initial tax rate on foreign assets, τ_0^x. However, once the value function from period one onwards is know it is straightforward to solve for the initial period allocation.

B.2 Markov-perfect Policies

B.2.1 Recursive Formulation

An often considered alternative to the assumption of full commitment to government policies in the dynamic Ramsey taxation literature is the other extreme of assuming no commitment. This appendix considers such Markov-perfect policies which can only be conditional on the payoff relevant state variables which will be here, a, τ^x, and g. The following analysis is along the line of the time-consistent public policy approach discussed in Klein, Krusell, and Ríos-Rull (the 2008). One way to rationalize their approach is to think of successive but identical governments that can only affect the policies of future government through the natural state variables but not through policy promises. Similar to Klein and Ríos-Rull (2003), I assume that governments set the tax on foreign asset income one period in advance to remain comparable to the case of Ramsey taxation. This avoids excessive taxation and τ^x becomes a state variable. To simplify the notation, I henceforth refer with τ to the tax on foreign asset income, τ^x. The recursive formulation of the government's decision problem can then be written as

$$\varphi(a, \tau, g) = \arg \max_{c, \ell, k, a', \tau'} u(c, \ell) + \beta \mathrm{E}_{g'|g} V(a', \tau', g')$$

subject to the resource constraint

$$0 \leq f(k, L - \ell) + (a - k) r^{\star} - \left[c + a' - (1-\delta) a + g + q(a)\right],$$

the private Euler equation

$$0 = \left[\beta \mathrm{E}_{g'|g} u_1 \left(c(a',\tau',g'),\ell(a',\tau',g')\right)\left[1+(1-\tau')r^\star-\delta-q_1(a')\right]-u_1(c,\ell)\right] a',$$

and the private budget constraint

$$0 = u_2(c,\ell)(L-\ell)+u_1(c,\ell)\Big[\left[1+(1-\tau)r^\star-\delta\right]a-\left[c+a'+q(a)\right]\Big],$$

where

$$\begin{aligned} V(a,\tau,g) &= u(c(a,\tau,g),\ell(a,\tau,g))+\beta \mathrm{E}_{g'|g} V(a',\tau',g'),\\ \varphi(a,\tau,g) &= \big\langle c(a,\tau,g),\ell(a,\tau,g),k(a,\tau,g),h^a(a,\tau,g),h^\tau(a,\tau,g)\big\rangle,\\ a' &= h^a(a,\tau,g),\ \tau' = h^\tau(a,\tau,g), \end{aligned}$$

with s_0, τ_0, a_0 and $\Gamma(g'|g)$ taken as given. This Markov-perfect formulation of the government problem is time-consistent because the future variables $\varphi(a',\tau',g')$ can only be affected by today's government through the manipulation of the endogenous state variables, a' and τ'. In next period's state (a',τ',g'), the next government will then find it optimal to set the expected policy $\varphi(a',\tau',g')$, even if it where allowed to deviate from it.

B.2.2 Optimal Tax on Physical Capital Rents

Independent of the assumption on the government's commitment technology, physical capital, k, is only present in the resource constraint of the government's decision problem. Thus, physical capital is optimally chosen to maximize resources in every period given the other variables and the corresponding equilibrium condition is still given by (2.14). No matter whether the government can commit to future policies or not, it is never optimal to distort the allocation of physical capital, thus $\tau^k = 0$.

B.2.3 Optimal Tax on Foreign Assets

The government's Euler equation can be written as

$$\begin{aligned}0 = \Bigg[& \beta \mathrm{E}_{g'|g} \lambda' \left[1 + r^\star - \delta - q_1(a')\right] - \lambda \\ & + \beta \psi \mathrm{E}_{g'|g} a' \left[\frac{\mathrm{d}u_1(c', \ell')}{\mathrm{d}a'} \left[1 + (1-\tau')r^\star - \delta - q_1(a')\right] - u_1\big(c', \ell'\big) q_{11}(a') \right] a' \\ & + \beta \mathrm{E}_{g'|g} \theta' u_1(c', \ell') \left[1 + (1-\tau')r^\star - \delta - q_1(a')\right] - \theta u_1(c, \ell) \Bigg],\end{aligned}$$

where λ, ψ and θ denote the multipliers on the resource constraint, the Euler equation, and the private budget constraint. In the jargon of Klein, Krusell, and Ríos-Rull (2008), this equation is called the *generalized* Euler equation, because not only the unknown functions enter the equilibrium conditions, but also their derivatives with respect to the endogenous states. So, by the very definition of the Markov-perfect equilibrium, it is only well defined if indeed the equilibrium functions are differentiable with respect to the states. In an interior deterministic steady-state the private Euler equation implies that

$$0 = \beta \left[1 + (1-\tau')r^\star - \delta - q_1(a)\right] - 1,$$

therefore, the optimal long-run tax rate on foreign asset income reads

$$\tau^x = \frac{\psi}{\lambda} a \left[u_1(c, \ell) q_{11}(a) - \frac{\mathrm{d}u_1(c, \ell)}{\mathrm{d}a} \right].$$

This shows that in the Markov-perfect equilibrium, the long-run optimal tax on foreign asset income will be positive even without the convex portfolio cost. The optimal time-consistent tax on foreign asset income is higher that in the long-run Ramsey equilibrium, because government's cannot commit not to tax predetermined assets in future periods.

Chapter 3

A Theory of Cross-Country Government Debt Distribution

This chapter is joint work with Zheng Song, Kjetil Storesletten, and Fabrizio Zilibotti. I am responsible for 50 percent of the work, the remaining half was contributed in equal parts by my co-authors.

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3.1 Introduction

There is a large variation in government debt policies across countries and over time. For instance, in a recent sample of its current 34 member countries, the OECD (2010a) reports that the average central government debt between 2000 and 2010 was as low as 3% of gross domestic product in Estonia, 4% in Luxembourg, and 8% in Australia. While the debt-output ratio was on average as high as 152% in Japan, 114% in Greece, and 100% in Italy over the same time period. Moreover, looking at a subsample[1] of 19 OECD countries for which Reinhart and Rogoff (2009) report gross central government debt-output ratios for the whole postwar period, there is also substantial mobility within the cross-country distribution of government debt over time.[2] For example, Japan that ends up in the top decile of the distribution in the 2000s started with a debt-output ratio in the second decile of the cross-country distribution 50 years earlier. Other countries like Australia and Ireland have moved down in the distribution over time, while Denmark and Norway have recovered from high debt levels in the 1970s and 1980s. Furthermore, the cross-country distribution of government debt in OECD countries has remained relatively stable in the postwar period and is not fanning out. Suggesting that the long-run debt levels are stationary.

While the large variation in the cross-country distribution of government debt could be driven by *ex ante* heterogeneity, the observed mobility within the distribution suggests that a substantial part of the variation also stems from idiosyncratic shocks that create *ex post* heterogeneity between countries. Addressing this pattern of the data, our theory will be flexible enough to allow for a variety of idiosyncratic shocks (political, preference, fiscal, demographic) that potentially affect the determination of government debt.

The discussion on the possible determinants of government debt has been revived with the seminal contribution of Barro (1979). Since then, the normative public finance literature has identified several important aspects of government debt. While Barro (1979) has argued that government debt only has a second-order role in smoothing tax distortions in response to fiscal shocks, Cukierman and Meltzer (1989) have emphasized that government debt can substitute for negative bequests and intergenerational transfers, and Aiyagari and McGrattan (1998) showed that public debt completes financial markets and loosens borrowing constraints. Battaglini and Coate (2008) on the other hand have empha-

[1] The countries in this subsample are Australia, Austria, Belgium, Canada, Chile, Denmark, Finland, France, Germany, Greece, Ireland, Japan, Norway, Portugal, Spain, Switzerland, Sweden, Turkey, and the United States (U.S.). Note that Chile (1983) and Turkey (1978, 1982) defaulted on their debt in the indicated years. For Hungary and Korea only part of the postwar period is covered. The United Kingdom only reports *net* central government debt, and the times series of Iceland, Italy, Netherlands, New Zealand, and Poland only cover gross *general* government debt.

[2] The deciles in the cross-country distribution for the complete set of countries are reported in Table C.1 of Appendix C.1.

sized that politico-economic forces distort the benevolent planner's motive to accumulate a buffer stock of assets (negative debt) to self-insure against fiscal shocks as was pointed out in Aiyagari et al. (2002). Persson and Svensson (1989) as well as Alesina and Tabellini (1990) have analyzed the political conflict arising from the fact that government debt shifts the burden of current government expenditure to future generations and crowds out future public good provision. Song, Storesletten, and Zilibotti (2012) have extended this political conflict to a fully dynamic general equilibrium setting.

In this paper we propose a dynamic political equilibrium model in which countries of an economically integrated world economy are subject to idiosyncratic shocks. The distribution of government debt in our model is purely driven by the history of realized shocks in each country, and we do not rely on *ex ante* country-specific differences that could also affect the distribution government debt. The proposed theory is flexible enough to incorporate several types of shocks, like political shifts, wars, or demographic transitions. We strengthen the view that politico-economic considerations are important to understand the distribution of government debt. Motivated by the empirical evidence that debt policy is correlated with the party ideology of governments on the left-right spectrum, we demonstrate that our model generates a reasonable fit to of the cross-country distribution of government debt observed in the data by allowing for idiosyncratic political shocks alone.

In the first part of the paper, we augment the dynamic politico-economic theory of Song, Storesletten and Zilibotti (2012) with idiosyncratic shocks at the country level and characterize the stationary equilibrium distribution across countries as was pioneered in Huggett (1993) and Aiyagari (1994) across workers for a given economy. We keep the model very general in that we allow for shocks along the political, fiscal or demographic dimension, and in the special case of the model where the labor supply is inelastic, we can characterize a quasi-stationary equilibrium of the model analytically and proof uniqueness. Assuming that left-leaning (right-leaning) governments are elected when the population puts a relatively high (low) weight on public goods, we can model political shocks as a change in the population's preference for the public goods.[3] Changes in the ideology of governments lead to changes in fiscal policy: right-wing governments run larger deficits and accumulate more debt because increasing debt today will partially finance a current tax break at the cost of crowding out future public good provision. Thus leftist governments are less eager to increase the debt compared to rightist governments.

[3]As discussed in Song, Storesletten and Zilibotti (2007), an alternative to assuming preference shocks would be to study a specification with cross-sectional wage heterogeneity (persistent over cohorts) and progressive taxation leading to an intra-generational conflict between the rich and the poor. Public good provision entails then a redistributive component: the poor want more government expenditure than the rich. In equilibrium, the level of government expenditure depends on the political clout of the poor relative to the rich. “Leftist” times are then periods when the poor have more political influence and can impose higher taxes and expenditure relative to “rightist” times.

In the second part of the paper we show that our hypothesis is supported by the data and provide evidence for the empirical correlation between debt policy and the left-right orientation of governments. We find the correlation in a panel of OECD countries as well as for the U.S. time series in separate for the postwar period. For example, with a Republican U.S. President in office the average increase in the debt-output ratio is about 1.2 percentage points per year higher compared to a Democrat U.S. President. The difference is statistically significant and robust to the inclusion of a business cycle measure and the demographic structure. Qualitatively similar results are obtained in a panel of OECD countries where however the measure of the political orientation of governments is a bit less clear cut than for the the U.S.. Moreover, confirming the results of Bohn (1998) we document evidence that the debt-output ratio is mean reverting which contradicts the hypothesis of Barro (1979) that debt growth should be independent of the debt-output ratio and therefore non-stationary. Finally, for the U.S. time-series we also find evidence that a Democrat President sets tax rates on average by around half a percentage point higher than a Republican President for a given debt-output ratio. A prediction that is consistent with our theoretical model presented in the first part.

In the final part of the paper, we show that with a parsimonious calibration of the stochastic process of the political shock, the model generates a reasonable fit to the cross-country distribution of government debt observed in the data. Thus, idiosyncratic political shocks alone are able to generate the large variation in government debt levels, even though, *ex ante* heterogeneity as well as other types of shocks could have complementary roles.

Our paper contributes to a broad literature on the politico-economic determinants of government debt. Among the theoretical contributions Cukierman and Meltzer (1989), Persson and Svensson (1989), Alesina and Tabellini (1990), and more recently, Battaglini and Coate (2008), Yared (2010), Azzimonti (2011), and Battaglini (2011) emphasize political conflict as a driving factor for public debt. Different from us, these papers focus on closed economies that could not explain the cross-country distribution of government debt. Song, Storesletten and Zilibotti (2012) consider an integrated world economy but do not consider a stochastic environment. On the empirical side several authors have tested the implications of the strategic debt models, albeit with mixed success. For example, Lambertini (2003) finds little support for the strategic use of budget deficits for the U.S. and in OECD panel data, while Pettersson-Lidbom (2001) finds significant support in data on Swedish municipalities.

A growing related politico-economic literature on time-consistent dynamic fiscal policy, where heterogeneous agents vote repeatedly on redistribution and taxation, includes Krusell et al. (1996), Krusell and Ríos-Rull (1999), Hassler et al. (2003), Hassler et al. (2005), Song (2011, 2012), and Klein, Krusell, and Ríos-Rull (2008). These papers are also methodologically similar to ours, although they assume a balanced government budget and do not deal with determinants of government debt.

The paper is organized as follows. In Section 3.2 we introduce the stochastic environment into the dynamic political general equilibrium model presented in Song, Storesletten and Zilibotti (2012) and characterize the stationary as well as a quasi-stationary equilibrium. In Section 3.3 we discuss two examples that admit an analytical solution of the quasi-stationary equilibrium. In Section 3.4 we provide empirical evidence for the correlation between the party ideology of governments and debt policy. In Section 3.5 we numerically solve for the stationary equilibrium of the model in the general case and show that the model is able to provide a reasonable fit to the empirical distribution of government debt. Section 3.6 concludes. Appendix C.1 contains the tables that are not in the main part of the paper. Appendix C.2 contains the details of the empirical analysis in Section 2. And Appendix C.3 contains the details of the model's calibration for the numerical analysis in Section 3.5.

3.2 Model

The model we present in this section is a stochastic version of the dynamic political equilibrium model of Song, Storesletten, and Zilibotti (2012). We will augment their deterministic multi-country setup with idiosyncratic shocks, such that the stationary equilibrium distribution of government debt allows for mobility of individual countries as observed in the data. This equilibrium concept was pioneered in Huggett (1993) and Aiyagari (1994) for the wealth distribution within a country. In a special case discussed in Section 3.3, where the labor supply of agents is inelastic, we are able to characterize a quasi-stationary equilibrium distribution of our model analytically.

3.2.1 Preferences

Song, Storesletten, and Zilibotti (2012) consider a unit measure of small open economies populated by a unit measure of overlapping generations of two-period-lived agents. Agents work in the first period, live off savings in the second period, and they have preferences over a private good, c, and a domestic public good, g. The expected utility of a young individual in country $j \in [0,1]$ born in period t is given by

$$U_{Y,j,t} = \log(c_{Y,j,t}) + \theta_{j,t} \log(g_{j,t}) + \beta \mathrm{E}_{t+1} \left(\log(c_{O,j,t+1}) + \lambda \theta_{j,t+1} \log(g_{j,t+1}) \right), \tag{3.1}$$

where β is the discount factor, and θ and $\lambda\theta$ capture the preferences for public goods relative to the private good for young and old agents, respectively. $\lambda > 1$ implies that old agents have a higher preference for the public good than young agents living in the same period. In the preference specification of Equation (3.1) we deviate from the setup in Song, Storesletten, and Zilibotti (2012) in that we allow for a stochastic environment. Correspondingly, E_{t+1} denotes the expectational operator conditional on the information available at date t. For tractability, we assume that the idiosyncratic shock denoted by $\tilde{\theta}_{j,t}$ follows a Markov process and is drawn from the finite-valued set Θ.[4] As a consequence, the stationary distribution of government debt will not be driven by *ex ante* heterogeneity, but by the *ex post* heterogeneity that is created by the history of shocks realization in each country over time. This allows the stationary debt level of individual countries to move within the stationary cross-country distribution.

3.2.2 Production Technology

The production function in each country is given by

$$Y_j = QK_j^{\alpha}H_j^{1-\alpha}, \alpha \in (0,1),$$

where K_j and H_j denote the aggregate capital and labor supply, respectively, in each country j. Labor and capital markets are competitive, factor prices are therefore given by the marginal product of labor and capital. Capital is perfectly mobile across countries and depreciates fully after one period. Let R denote the world interest rate and w_j the workers wage rate in each country. In equilibrium, the wage will be a function of the world interest rate so that the local market for labor clears,

$$w_j = W(R) \equiv (1-\alpha)\, Q^{1/(1-\alpha)} \left(\frac{\alpha}{R}\right)^{\alpha/(1-\alpha)},$$

implying that the wage rate is identical across countries. Similarly, the country-specific capital stock will be a function of the world interest rate and the local labor supply,

$$K_j = K\left(R, H_j\right) \equiv \left(\frac{\alpha Q}{R}\right)^{1/(1-\alpha)} H_j$$

[4] In Section 3.5, we focus the analysis on the case where only the relative preference for public goods is stochastic, such that $\tilde{\theta}_{j,t} = \theta_{j,t}$. Based on the empirical evidence presented in Section 3.4, we think that is a particularly interesting case. In this section however, we keep the analysis very general and present a framework where also part of the government expenditures or the demographic structure could be stochastic, for example.

Henceforth, we switch to recursive notation and drop country indices whenever this is no source of confusion.

3.2.3 Labor Supply and Savings

Workers can choose to supply an amount h of their unit time labor endowment to market production at the wage rate, $W(R)$. The remaining time, $1-h$, is then devoted to home production[5] yielding $F(h)$. Since the government can only tax market production, the labor tax, τ, will distort the labor supplied to the market. The country-specific decision rules of agents are given by the labor supply function,

$$H(\tau, R) = \arg\max_{0\leq h\leq 1} \ (1-\tau)hW(R) + F(h) + \frac{F(0)}{R},$$

where it will also be convenient to define the discounted lifetime income from labor as

$$A(\tau, R) \equiv (1-\tau)H(\tau, R)W(R) + F(H(\tau, R)) + \frac{F(0)}{R}.$$

The logarithmic utility implies that young agents consume a fraction $(1+\beta)^{-1}$ of their discounted lifetime income, therefore, the savings function reads

$$\begin{aligned} S(\tau, R) &\equiv (1-\tau)W(R)H(\tau, R) + F(H(R, \tau)) - \frac{1}{1+\beta}A(\tau, R) \\ &= \frac{\beta}{1+\beta}A(\tau, R) - \frac{F(0)}{R}. \end{aligned}$$

Ignoring constant terms that do not depend on government policy, the discounted utilities of the young and the old agents in a country are then given by

$$U_Y(\tau, g, g') = (1+\beta)\log\left(A\left(\tau, R\right)\right) + \theta\log\left(g\right) + \beta\lambda \mathrm{E}_{\tilde{\theta}'|\tilde{\theta}}\theta'\log\left(g'\right) \tag{3.2}$$

$$U_O(g) = \log\left(A\left(\tau_{-1}, R\right)\right) + \lambda\theta\log\left(g\right), \tag{3.3}$$

which illustrates that the current tax policy, τ, does not affect old agent's utility which only depends on the previous tax rate, τ_{-1}.

3.2.4 Government

The domestic fiscal policy is determined through repeated elections of governments. The current government inherits a debt level, b, from the previous government, and chooses the

[5] The regularity conditions for home production are that $F(1)=0$, the first derivative is strictly positive, and the second and third derivative are weakly positive.

current labor income tax, τ, the future debt level, b', and the level of public goods, g, as announced in the electoral campaign. Each government is subject to the budget constraint

$$b' = g + z + Rb - \tau W(R) H(\tau, R),$$

and cannot borrow in the international capital market beyond the present value of the maximum future expected primary surplus,

$$\bar{b}(R) \equiv \max_{0\leq\tau\leq 1} \frac{\tau H(\tau, R) W(R) - \bar{z}}{R-1},$$

where $\bar{z}$ denotes the highest realization of the non-productive and idiosyncratic fiscal expenditure shock, z. Finally, we model the political mechanism as a probabilistic voting model based on Lindbeck and Weibull (1987) and Persson and Tabellini (2000, pp. 54-58). Formally, the political objective function is given by

$$U(\tau, g, g') = (1-\omega)\, U_Y(\tau, g, g') + \omega U_O(g),$$

where ω is the relative weight on old agents.[6] Using Equations (3.2) and (3.3), the political objective function can be written as

$$\begin{aligned} U(\tau, g, g') = {} & [1 + \omega(\lambda - 1)]\theta \log(g) + (1-\omega)(1+\beta) \log(A(\tau, R)) \\ & + (1-\omega)\, \beta\lambda \mathrm{E}_{\tilde{\theta}'|\tilde{\theta}} \theta' \log(g') . \end{aligned}$$

3.2.5 Stationary Equilibrium

In the world equilibrium, the distribution of debt and wealth levels across countries are in general state variables that determine the world interest rate and therefore the individual policy rules in equilibrium. Here instead, we focus on stationary equilibria where the world interest rate is constant and the individual policy rules are functions of the country-specific state variables and the constant world interest rate only. This equilibrium concept was pioneered in Hugget (1993) and Aiyagari (1994).

In each country, fiscal policy is determined by the dynamic game between successive generations of voters. Ruling out reputational mechanisms, we restrict attention to Markov-perfect equilibria (MPE) where voters can condition their strategies on payoff-relevant state variables only. Since an individual country has no mass, the policy of an individual country does not affect the stationary distribution of debt and wealth, so voters take the equilibrium

[6] A microfoundation of this political objective function is provided in Appendix B of Song, Storesletten, and Zilibotti (2012).

interest rate sequence as given. Moreover, since private wealth does not affect the political preference of old voters, the debt level b and the realization of the idiosyncratic shock, $\tilde{\theta}$, are the only domestic payoff-relevant state variables.[7] The individual state of a country is then given by its government debt $b \in \left[\underline{b}, \bar{b}(R)\right]$ and its type $\tilde{\theta} \in \Theta$.[8]

Let χ be a probability measure defined on (Ω, Σ_Ω), where $\Omega \equiv \left[\underline{b}, \bar{b}(R)\right] \times \Theta$ is the individual state space, and Σ_Ω is the Borel σ-algebra on Ω (i.e., the set of all possible subsets of Ω). Thus, for any set $s \in \Sigma_\Omega$, the measure of countries whose individual state vectors lie in the set s is given by $\chi(s)$.

Definition 1 *A stationary Markov-perfect political equilibrium is an interest rate R^*, a probability measure χ^*, and a set of functions $\langle B, G, T \rangle$, where $B : \Omega \times \mathbb{R}^+ \to \mathbb{R}$ is a debt rule, $b' = B\left(b, \tilde{\theta}, R\right)$, $G : \Omega \times \mathbb{R}^+ \to [0, \infty)$ is a government expenditure rule, $g = G\left(b, \tilde{\theta}, R\right)$, and $T : \Omega \times \mathbb{R}^+ \to [0, 1]$ is a tax rule, $\tau = T\left(b, \tilde{\theta}, R\right)$, such that*

(1) Fiscal policy is optimal and time consistent:

$$\left\langle B\left(b, \tilde{\theta}, R\right), G\left(b, \tilde{\theta}, R\right), T\left(b, \tilde{\theta}, R\right)\right\rangle = \arg\max_{b', g, \tau} \; U(\tau, g, g')$$

subject to[9]

$$\begin{aligned} &b' = g + z + R^* b - \tau W(R^*) H(\tau, R^*) \\ &g' = G\left(b', \tilde{\theta}', R^*\right) \\ &b' \in \left[\underline{b}, \bar{b}(R^*)\right], \quad \tau \in [0, 1], \quad g \geq 0. \end{aligned}$$

(2) The world asset market clears:

$$\begin{aligned} &\int_\Omega \left\{ S\left(T\left(b, \tilde{\theta}, R^*\right), R^*\right) - B\left(b, \tilde{\theta}, R^*\right) \right\} d\chi^*\left(b, \tilde{\theta}\right) \\ &= \int_\Omega \sum_{\tilde{\theta}'} \Gamma\left(\tilde{\theta}' \mid \tilde{\theta}\right) K\left(T\left(B\left(b, \tilde{\theta}, R^*\right), \tilde{\theta}', R^*\right), R^*\right) d\chi^*\left(b, \tilde{\theta}\right), \end{aligned}$$

[7]The fact that the private wealth of the old agents does not matter for the political equilibrium is due to three assumptions made above: (i) the country is a small open economy, (ii) capital and wealth are not taxed (although the aggregate world capital stock is obviously relevant), and (iii) preferences over public goods and private goods are additively separable.

[8]For standard technical reasons, we impose a lower bound on debt, $\underline{b}$. Such lower bound must be chosen sufficiently low so as not to be binding in equilibrium.

[9]The constraint on b' above implies that the equilibrium fiscal policy satisfies the government's budget constraint,

$$B\left(b, \tilde{\theta}, R^*\right) = G\left(b, \tilde{\theta}, R^*\right) + z + R^* b - T\left(b, \tilde{\theta}, R^*\right) W(R^*) H\left(T\left(b, \tilde{\theta}, R^*\right), R^*\right).$$

where the left-hand side is aggregate savings minus bond holdings and the function

$$K\left(T\left(B\left(b,\tilde{\theta},R^{*}\right),\theta',R^{*}\right),R^{*}\right)$$

is the next-period capital stock for a country with current state $\left(b,\tilde{\theta}\right)$ *and a realization* $\tilde{\theta}'$ *next period given the Markov transition probability,* $\Gamma\left(\tilde{\theta}' \mid \tilde{\theta}\right)$.

(3) *The probability measure* χ^{*} *is stationary:*

$$\chi^{*}(s)=\int_{\Omega} I\left\{\left(B\left(b,\tilde{\theta},R^{*}\right),\tilde{\theta}'\right)\in s\right\} d\chi^{*}\left(b,\tilde{\theta}\right) \qquad \textit{for all } s\in\Sigma_{\Omega},$$

where I *is an indicator function that takes the value unity if a country with current state* $\left(b,\tilde{\theta}\right)$ *is in the set* s *next period, and zero otherwise.*

For future reference, we will also define a quasi-stationary equilibrium where we relax condition (3) of Definition 1 in that the we require only the average debt level conditional on $\tilde{\theta}$ to be constant over time while the probability measure χ is allowed to be non-stationary. In Section 3.3, we will illustrate two analytical examples that satisfy this weaker equilibrium concept, where the debt distribution as a whole can be ever-expanding, but the mean debt level conditional on the realized shock $\tilde{\theta}$ is constant over time.

To this point, let us redefine the individual state space to $\hat{\Omega}\equiv\left(-\infty,\bar{b}(R)\right]\times\Theta$ and let $\Sigma_{\hat{\Omega}}$ be the Borel σ-algebra on $\hat{\Omega}$. Since we allow for ever-expanding distributions, we also have to remove the lower bound on government debt, $b'\in(-\infty,\bar{b}(R)]$[10].

Definition 2 *A quasi-stationary Markov-perfect political equilibrium is a constant interest rate* R^{*}*, a sequence of probability measures* $\{\chi_t\}_{t=0}^{\infty}$ *defined on* $(\hat{\Omega},\Sigma_{\hat{\Omega}})$*, and a set of functions* $\langle B,G,T\rangle$*, where* $B:\hat{\Omega}\times\mathbb{R}^{+}\to\mathbb{R}$ *is a debt rule,* $b'=B\left(b,\tilde{\theta},R\right)$*,* $G:\hat{\Omega}\times\mathbb{R}^{+}\to[0,\infty)$ *is a government expenditure rule,* $g=G\left(b,\tilde{\theta},R\right)$*, and* $T:\hat{\Omega}\times\mathbb{R}^{+}\to[0,1]$ *is a tax rule,* $\tau=T\left(b,\tilde{\theta},R\right)$*, such that equilibrium conditions (1) and (2) of Definition 1 are satisfied for* $b'\in(-\infty,\bar{b}(R)]$ *and, in addition, the following two conditions hold:*

(1) *The sequence of probability measures* $\{\chi_t\}_{t=0}^{\infty}$ *is consistent with the individual policy rule:*

$$\chi_{t+1}(s)=\int_{\hat{\Omega}} I\left\{\left(B\left(b,\tilde{\theta},R^{*}\right),\theta'\right)\in s\right\} d\chi_{t}\left(b,\tilde{\theta}\right), \quad \textit{for all } s\in\Sigma_{\Omega}.$$

[10] Alternatively, we could define a sequence of lower bounds becoming increasingly wide and growing at the rate of the stationary distributions dispersion.

(2) The conditional average debt is time invariant:

$$E_{\chi_{t+1}}\left\{b \mid \tilde{\theta}_j\right\} = E_{\chi_t}\left\{b \mid \tilde{\theta}_j\right\}, \quad \forall \tilde{\theta}_j \in \Theta, t \geq 0,$$

implying that

$$\int_{\hat{\Omega}} b \cdot I\left\{\tilde{\theta} = \tilde{\theta}_j\right\} d\chi_{t+1}\left(b, \tilde{\theta}\right) = \int_{\hat{\Omega}} b \cdot I\left\{\tilde{\theta} = \tilde{\theta}_j\right\} d\chi_t\left(b, \tilde{\theta}\right), \quad \forall \tilde{\theta}_j \in \Theta, t \geq 0.$$

3.3 Inelastic Labor Supply

In this section, we focus the analysis on the special case where young agents supply their unit endowment of labor inelastically to the market and old agents do not involve in any non-market production. More specifically, we set $H(\tau, R) = 1$ and $A(\tau, R) = (1-\tau) W(R)$. Moreover, we abstract from the government expenditure shock, z, while the relative preference for public goods, θ, or the relative political weight of the old, ω, could still be stochastic. This restrictions will allow for an analytical solution of the Markov-perfect equilibrium characterized in condition (1) of Definition 1. In addition, we set the preference weight for the old agents to $\lambda = 1$, which is not crucial to obtain an analytical solution of the quasi-stationary equilibrium but simplifies terms somewhat. Finally, because tax policy is non-distortionary, maximum tax revenue is raised at $\bar{\tau} = 1$ which implies a borrowing limit of $\bar{b} = W(R)/(R-1)$.

3.3.1 Markov-Perfect Political Equilibrium

Taking as given the world interest rate R, each country's Markov-perfect political equilibrium (see condition (1) of Definition 1) is characterized by a system of three functional equations:

$$\frac{(1-\omega)(1+\beta)}{W(R)\left(1 - T\left(b, \tilde{\theta}, R\right)\right)} = \frac{\theta}{G\left(b, \tilde{\theta}, R\right)} \tag{3.4}$$

$$\frac{\theta}{G\left(b, \tilde{\theta}, R\right)} = -(1-\omega)\beta \mathrm{E}_{\tilde{\theta}'|\tilde{\theta}} \frac{\theta' G_1\left(B\left(b, \tilde{\theta}, R\right), \tilde{\theta}', R\right)}{G\left(B\left(b, \tilde{\theta}, R\right), \tilde{\theta}', R\right)} \tag{3.5}$$

$$B\left(b, \tilde{\theta}, R\right) = G\left(b, \tilde{\theta}, R\right) + Rb - T\left(b, \tilde{\theta}, R\right) W(R), \tag{3.6}$$

where $G_1(\cdot)$ denotes the partial derivative of $G(\cdot)$ with respect to the debt level which is the unknown functions first argument. The first equilibrium condition in Equation (3.4) captures the intratemporal trade-off between the marginal cost of taxation for the young agents and the marginal benefit of public good provision for both types of agents. This trade-off reflects the political conflict between young and old voters: as old agents do not suffer from current taxation, they would vote for the maximal tax rate $\bar{\tau} = 1$ on the young agents if holding all the political power. The second equilibrium condition in Equation (3.5) is a generalized Euler equation for public good consumption incorporating the disciplining effect imposed by the young voters who anticipate that debt accumulation will crowd out future public good provision. This disciplining effect is more pronounced the higher the expected marginal utility of the public good is for the next period.

We guess and verify that the equilibrium policy functions are linear in the current debt level, b. Substituting this guesses into (3.4)-(3.6) and solving for the undetermined coefficients yields the following result.

Proposition 1 *The Markov-perfect political equilibrium is characterized by the following policy functions:*

$$B\left(b, \tilde{\theta}, R\right) = \bar{b} - \gamma\left(\tilde{\theta}\right) R\left(\bar{b} - b\right), \tag{3.7}$$

$$G\left(b, \tilde{\theta}, R\right) = \frac{\theta\gamma\left(\tilde{\theta}\right)}{(1-\omega)\,\beta E_{\theta'|\theta}\theta'} R\left(\bar{b} - b\right), \tag{3.8}$$

$$T\left(b, \tilde{\theta}, R\right) = 1 - \frac{(1+\beta)\,\gamma\left(\tilde{\theta}\right)}{\beta E_{\theta'|\theta}\theta'} \frac{R}{W(R)}\left(\bar{b} - b\right), \tag{3.9}$$

where $\bar{b} \equiv W/\left(R-1\right)$ *and,*

$$\gamma\left(\tilde{\theta}\right) \equiv \frac{(1-\omega)\,\beta E_{\theta'|\theta}\theta'}{\theta + (1-\omega)\left[1 + \beta(1 + E_{\theta'|\theta}\theta')\right]} > 0.$$

Note that only the conditional mean of the relative preference for the public good enters the equilibrium functions, while the higher moments as well as the other stochastic variables cancel out due to the logarithmic specification of preferences. As long as the persistence of the preference shock is high enough $\gamma(\tilde{\theta})$ is increasing in θ, which implies that a higher weight on public goods reduces debt accumulation. On the other hand $\gamma(\tilde{\theta})$ is decreasing in ω, reflecting that debt accumulation is more pronounced if the old voters have more political power. Moreover, the provision of the public goods is falling in the debt level while the opposite holds for taxes. Because the transition probabilities of the stochastic Markov process enter the equilibrium functions $G(\cdot)$, $B(\cdot)$ and $T(\cdot)$ through the conditional expectation operator that is incorporated in $\gamma(\tilde{\theta})$, the strategic motive for debt accumulation that has been proposed in Persson and Svensson (1989) is also present in our model. For instance, a government with a relatively low taste for public goods will issue more debt if it anticipates to be replaced by a government with a stronger taste for public expenditures in the next period.

3.3.2 Quasi-Stationary Equilibrium

As a next step, we consider the general equilibrium determination of the world interest rate, R, and pin down the quasi-stationary equilibrium. An important implication of the linear law of motion of debt in Equation (3.7) is that if we consider a set of countries with a realization $\tilde{\theta}_i$ today, their average debt tomorrow only depends on their cross-sectional average debt today,

$$\mathrm{E}_{\chi_t}\left\{B\left(b,\tilde{\theta},R\right)|\tilde{\theta}_i\right\} = \bar{b} - \gamma\left(\tilde{\theta}_i\right)R\left(\bar{b} - \mathrm{E}_{\chi_t}\left\{b|\tilde{\theta}_i\right\}\right).$$

This follows from the fact that all countries with $\tilde{\theta} = \tilde{\theta}_i$ have the same proportional change in their debt, $\bar{b} - b$. Let π_{ij} denote the constant probability of transiting from state $\tilde{\theta}_i$ to state $\tilde{\theta}_j$ in any period. Moreover, π_j is the unconditional stationary probability for a country to be of type $\tilde{\theta}_j$. Then tomorrow's average debt conditional on $\tilde{\theta}_j$ is a weighted sum of tomorrow's average debt conditional on the realization of today's θ_i consistent with the exogenous transition probabilities,

$$\begin{aligned}
\mathrm{E}_{\chi_{t+1}}\left\{b|\tilde{\theta}_j\right\} &= \sum_{i=1}^{N}\frac{\pi_{ij}\pi_i}{\pi_j}\mathrm{E}_{\chi_t}\left\{B\left(b,\tilde{\theta},R\right)|\tilde{\theta}_i\right\} \\
&= \bar{b} - \sum_{i=1}^{N}\frac{\pi_{ij}\pi_i}{\pi_j}\gamma(\tilde{\theta}_i)R\left(\bar{b} - \mathrm{E}_{\chi_t}\left\{b|\tilde{\theta}_i\right\}\right).
\end{aligned}$$

Suppose that there exists a quasi-stationary equilibrium such that the cross-sectional average debt level conditional on $\tilde{\theta}_j$, denoted by $b_j^* = E_{\chi_{t+1}}\{b|\tilde{\theta}_j\} = E_{\chi_t}\{b|\tilde{\theta}_j\}$, is time-invariant for all j (this guess must be confirmed), then constant conditional means imply that[11]

$$R^{-1}\left(\bar{b} - b_j^*\right) = \sum_{i=1}^{N}\frac{\pi_{ij}\pi_i}{\pi_j}\gamma\left(\tilde{\theta}_i\right)\left(\bar{b} - b_i^*\right), \quad \forall j. \tag{3.10}$$

The N equations in (3.10) together with the world capital market clearing condition of Definition 1 pin down the N unknown conditional debt levels b_j^* and the world interest rate R. Equation (3.10) can be written in matrix form

$$\mathbf{Av} = R^{-1}\mathbf{v},$$

[11]Note that if the types are persistent, $\pi_{jj} = 1$, either (or both) of the following must be true: (i) $\gamma(\tilde{\theta}_j) = 1$; (ii) $b_j^* = \bar{b}$. This is the special case analyzed in Song, Storesletten, and Zilibotti (2012).

where

$$
\mathbf{A} \equiv \begin{bmatrix} \pi_{11}\gamma\left(\tilde{\theta}_1\right) & \pi_{21}\frac{\pi_2}{\pi_1}\gamma\left(\tilde{\theta}_2\right) & \dots & \pi_{N1}\frac{\pi_N}{\pi_1}\gamma\left(\tilde{\theta}_N\right) \\ \pi_{12}\frac{\pi_1}{\pi_2}\gamma\left(\tilde{\theta}_1\right) & \pi_{22}\gamma\left(\tilde{\theta}_2\right) & \dots & \pi_{N2}\frac{\pi_N}{\pi_2}\gamma\left(\tilde{\theta}_N\right) \\ \pi_{13}\frac{\pi_1}{\pi_3}\gamma\left(\tilde{\theta}_1\right) & \pi_{23}\frac{\pi_2}{\pi_3}\gamma\left(\tilde{\theta}_2\right) & \dots & \pi_{N3}\frac{\pi_N}{\pi_3}\gamma\left(\tilde{\theta}_N\right) \\ \vdots & \vdots & \dots & \vdots \\ \pi_{1N}\frac{\pi_1}{\pi_N}\gamma\left(\tilde{\theta}_1\right) & \pi_{2N}\frac{\pi_2}{\pi_N}\gamma\left(\tilde{\theta}_2\right) & \dots & \pi_{NN}\gamma\left(\tilde{\theta}_N\right) \end{bmatrix}, \quad \mathbf{v} \equiv \begin{bmatrix} \bar{b} - b_1^* \\ \bar{b} - b_2^* \\ \bar{b} - b_3^* \\ \vdots \\ \bar{b} - b_N^* \end{bmatrix}.
$$

are non-negative matrices. Note that R^{-1} is an eigenvalue of $\mathbf{A}$ with the associated eigenvector $\mathbf{v}$. Assuming that all entries of matrix $\mathbf{A}$ are strictly positive, we can now invoke the Perron-Frobenius theorem (see Meyer (2000), and references therein) in linear algebra to establish that (i) there exists one and only one eigenvalue of $\mathbf{A}$ associated with a strictly positive eigenvector $\mathbf{v}$, (ii) this is the largest eigenvalue. In particular, our solution is the smallest root of R, since the eigenvalue is R^{-1}. Thus, Let p be the largest eigenvalue of the matrix $\mathbf{A}$ and let $\mathbf{v} = (v_1, v_2, ..., v_N)'$ be its associated eigenvector (defined up to positive multiples). Then the interest rate $R = 1/p$ and the debt distribution, $\bar{b} - b_j^* = v_j > 0$ $\forall j$, normalized by the world capital market clearing condition is the unique allocation that satisfies the conditions for a quasi-stationary equilibrium.

In what follows we provide two analytical examples that illustrate the quasi-stationary equilibrium concept. To simplify notation we assume - without loss of generality - that θ is the only idiosyncratic shock in the economy such that $\tilde{\theta} = \theta$.

3.3.3 Example: $\pi_{jj} = 1/N$

Assuming that the realizations of the relative preference for the public good θ are independent and identically distributed, then the characterization of the quasi-stationary equilibrium is particularly simple. First, all limiting and transition probabilities are equal to $1/N$. Therefore Equation (3.10) becomes

$$
R^{-1}\left(\bar{b} - b_j^*\right) = \frac{1}{N}\sum_{i=1}^{N}\gamma\left(\theta_i\right)\cdot\left(\bar{b} - b_i^*\right),
$$

implying that all conditional debt levels must be identical, $\left(\bar{b} - b_j^*\right) = \left(\bar{b} - b^*\right)$. Furthermore, this also implies that the average growth rate of the debt level is equal to one,

$$
1 = \frac{1}{N}\sum_{i=1}^{N} R\gamma\left(\theta_i\right).
$$

which in turn yields an equation that pins down the stationary world interest rate R^* when we substitute in the expression for $\gamma(\theta_i)$,

$$R^* = \left(\frac{1}{N}\sum_{i=1}^{N}\gamma(\theta_i)\right)^{-1} = \left(\frac{1}{N}\sum_{i=1}^{N}\frac{\beta(1-\omega)\theta_i}{\omega\theta_i + (1-\omega)(1+\theta_i)(1+\beta)}\right)^{-1}.$$

We are left to pin down the stationary average debt level with the world capital market clearing condition. Because $H(\tau, R^*) = 1$, savings are given by

$$S(b,\theta,R^*) = \frac{\beta}{1+\beta}(1-T(b,\theta,R^*))W(R^*) = \frac{\gamma(\theta)}{\theta}R^*(\bar{b}-b),$$

and the capital stock is determined as

$$K(R^*) = \left(\frac{\alpha Q}{R^*}\right)^{1/(1-\alpha)}.$$

Because all the conditional average debt levels are identical, the world capital market clearing condition then reads

$$\frac{1}{N}\sum_{j=1}^{N}\frac{\gamma(\theta_j)R^*}{\theta_j}(\bar{b}-b^*) = \bar{b} - \frac{1}{N}\sum_{j=1}^{N}\gamma(\theta_j)R^*(\bar{b}-b^*) + K(R^*),$$

such that the stationary average debt level is given by

$$\bar{b}-b^* = \left[\bar{b} + \left(\frac{\alpha Q}{R^*}\right)^{1/(1-\alpha)}\right]\frac{1}{R^*}\left(\frac{1}{N}\sum_{j=1}^{N}\gamma(\theta_j)(1+1/\theta_j)\right)^{-1} > 0.$$

3.3.4 Example: $N = 2$

Considering a two-state Markov process with persistence $\pi_{jj} = \rho \in (1/2, 1)$ is another tractable case. Because the transition probabilities are symmetric across realizations, the unconditional stationary probabilities will be $\pi_j = 1/2$ and Equation (3.10) becomes

$$\begin{aligned} R^{-1}\left(\bar{b}-b_1^*\right) &= \rho\gamma(\theta_1)\left(\bar{b}-b_1^*\right) + (1-\rho)\gamma(\theta_2)\left(\bar{b}-b_2^*\right) \\ R^{-1}\left(\bar{b}-b_2^*\right) &= (1-\rho)\gamma(\theta_1)\left(\bar{b}-b_1^*\right) + \rho\gamma(\theta_2)\left(\bar{b}-b_2^*\right). \end{aligned}$$

Rearranging this system of equations yields

$$\gamma(\theta_1)R = \frac{1-\rho\gamma(\theta_2)R}{\rho+(1-2\rho)\gamma(\theta_2)R} \tag{3.11}$$

$$\bar{b}-b_2^* = \frac{1-\rho\gamma(\theta_1)R}{(1-\rho)\gamma(\theta_2)R}\left(\bar{b}-b_1^*\right). \tag{3.12}$$

Equation (3.11) pins down the two roots for the world interest rate $R^\star$ and Equation (3.12) shows that only the root associated with $\rho\gamma(\theta_1)R^* < 1$ is consistent with a quasi-stationary equilibrium (yields positive conditional mean debt levels, $\bar{b}-b$).

Lemma 1 *Suppose $\rho \in (1/2, 1)$. Equation (3.11) has two real positive roots. One and only one root is consistent with $\rho\gamma(\theta_1)R < 1$, and hence with both $b_1^* < \bar{b}$ and $b_2^* < \bar{b}$. This is given by*

$$R^* = \frac{\rho(\gamma(\theta_1)+\gamma(\theta_2)) - \sqrt{\rho^2(\gamma(\theta_1)+\gamma(\theta_2))^2 - 4(2\rho-1)\gamma(\theta_1)\gamma(\theta_2)}}{2(2\rho-1)\gamma(\theta_1)\gamma(\theta_2)}.$$

Proof: Equation (3.11) can be reformulated as the quadratic equation

$$0 = (1-2\rho)\gamma(\theta_1)\gamma(\theta_2)R^2 + \rho(\gamma(\theta_1)+\gamma(\theta_2))R - 1,$$

yielding the two roots

$$R^* = \frac{\rho(\gamma(\theta_1)+\gamma(\theta_2)) \pm \sqrt{\rho^2(\gamma(\theta_1)+\gamma(\theta_2))^2 - 4(2\rho-1)\gamma(\theta_1)\gamma(\theta_2)}}{2(2\rho-1)\gamma(\theta_1)\gamma(\theta_2)}.$$

The two roots are positive and real if and only if

$$\rho^2(\gamma(\theta_1)+\gamma(\theta_2))^2 - 4(2\rho-1)\gamma(\theta_1)\gamma(\theta_2) \geq 0. \tag{3.13}$$

Note that

$$4(2\rho-1)\gamma(\theta_1)^2\frac{(1-\rho)^2}{\rho^2} = \min_{\gamma(\theta_2)} \rho^2(\gamma(\theta_1)+\gamma(\theta_2))^2 - 4(2\rho-1)\gamma(\theta_1)\gamma(\theta_2) > 0.$$

Thus, for any given $\gamma(\theta_1)$, $\gamma(\theta_2)$, and $\rho \in (1/2, 1)$ the condition stated in Equation (3.13) is satisfied. Finally, according to the Perron-Frobenius theorem, only the smallest positive and real root for R^* will satisfy $b_1^* < \bar{b}$ and $b_2^* < \bar{b}$. ■

Remark 1 *The two extreme cases $\rho = 1/2$ and $\rho = 1$ are noteworthy. As $\rho \to 1/2$, the* *root tends to the iid case with $N = 2$*

$$R^* = \left(\frac{1}{2}\sum_{i=1}^{2}\gamma(\theta_i)\right)^{-1}.$$

When $\rho \to 1$, the root tends to the deterministic case described in Song, Storesletten, and Zilibotti (2012) for $N = 2$

$$R^* = \gamma(\theta_2)^{-1}.$$

Proof: If $\rho \to 1/2$, using Hopital's rule

$$\begin{aligned}
\lim_{\rho\to 1/2} R^* &= \lim_{\rho\to 1/2}\frac{\gamma(\theta_1)+\gamma(\theta_2) - \frac{2\rho(\gamma(\theta_1)+\gamma(\theta_2))^2 - 8\gamma(\theta_1)\gamma(\theta_2)}{2[\rho^2(\gamma(\theta_1)+\gamma(\theta_2))^2 - 4(2\rho-1)\gamma(\theta_1)\gamma(\theta_2)]^{1/2}}}{4\gamma(\theta_1)\gamma(\theta_2)}\\
&= \frac{\gamma(\theta_1)+\gamma(\theta_2) - (\gamma(\theta_1)+\gamma(\theta_2))^{-1}\left[(\gamma(\theta_1)+\gamma(\theta_2))^2 - 8\gamma(\theta_1)\gamma(\theta_2)\right]}{4\gamma(\theta_1)\gamma(\theta_2)}\\
&= \left(\frac{1}{2}\sum_{i=1}^{2}\gamma(\theta_i)\right)^{-1}.
\end{aligned}$$

If $\rho \to 1$, then

$$\lim_{\rho\to 1} \pm\sqrt{\rho^2(\gamma(\theta_1)+\gamma(\theta_2))^2 - 4(2\rho-1)\gamma(\theta_1)\gamma(\theta_2)} = \pm(\gamma(\theta_2)-\gamma(\theta_1)),$$

where only the negative root is consistent with $\lim_{\rho\to 1}\rho\gamma(\theta_1)R^* < 1$. The world interest rate is then given by

$$\lim_{\rho\to 1} R^* = \gamma(\theta_2)^{-1}.$$

■

To close the system, we use the asset market equilibrium condition. Since the probability transition matrix is symmetric and the limiting probability will be 1/2 for each type θ the asset market clearing condition reads

$$\frac{1}{2}\sum_{j=1}^{2}\frac{\gamma(\theta_j)}{\theta_j}R^*(\bar{b}-b_j^*) = \bar{b} - \frac{1}{2}\sum_{j=1}^{2}\gamma(\theta_j)R^*(\bar{b}-b_j^*) + K(R^*).$$

Using Equations (3.11) and (3.12) to reduce $\bar{b} - b_2^*$, the market clearing condition can be rewritten as

$$\bar{b} - b_1^* = \left(\bar{b} + K(R^*)\right) \frac{2}{R^*} \left(\gamma(\theta_1)(1 + 1/\theta_1)) + \frac{(1 + 1/\theta_2)(1 - \rho)^2 \gamma(\theta_2)^2}{(\rho + (1 - 2\rho)\gamma(\theta_2)R^*)(1 - \rho)\gamma(\theta_2)} \right)^{-1} .$$

Note that for $\rho \to 1/2$ the expression again converges to the iid case studied previously.

3.3.5 Ever-Growing Dispersion

In the two analytical examples of this section we have characterized a quasi-stationary equilibrium instead of stationary equilibrium because only the first moment but not the higher moments of the debt distribution are stationary. In fact, the distribution of debt is ever-expanding in the case with the inelastic labor supply. To see this, note that the logarithm of $\left(\bar{b} - b\right)$ in each country follows a random walk. Consider a country starting with $\left(\bar{b} - b_0\right)$. The period-$t$ debt of such a country is then

$$\left(\bar{b} - b_{t+1}\right) = \gamma\left(\theta_t\right) R^\star \left(\bar{b} - b_t\right) = \left(\prod_{k=0}^{t} \gamma\left(\theta_k\right) R^* \right) \left(\bar{b} - b_0\right) ,$$

where $\{\theta_k\}_{k=0}^t$ is the realized sequence of draws of θ for the considered country. Taking the natural logarithm yields

$$\log\left(\bar{b} - b_{t+1}\right) = \sum_{k=0}^{t} \log\left(\gamma\left(\theta_k\right) R^*\right) + \log\left(\bar{b} - b_0\right) .$$

The cross-sectional variance of debt in period t then reveals that the dispersion of debt is ever-increasing with t as long as the distribution of types θ is non-degenerate,

$$\begin{aligned} \mathrm{Var}\left[\log\left(\bar{b} - b_t\right)\right] &= \sum_{k=0}^{t-1} \mathrm{Var}\left[\log\gamma\left(\theta_k\right)\right] + \mathrm{Var}\left[\log\left(\bar{b} - b_{i,0}\right)\right] \\ &= t \cdot \mathrm{Var}\left[\log\gamma\left(\theta_k\right)\right] + \mathrm{Var}\left[\log\left(\bar{b} - b_{i,0}\right)\right] . \end{aligned}$$

In conclusion, the equilibrium with the inelastic labor supply is quasi-stationary in the sense that the world interest rate and the distribution of the average debt levels conditional on θ are time invariant. However, the cross-sectional distribution of debt is fanning out: the cross-sectional variance and the cross-sectional variance increases linearly over time. Note that since the debt level is bounded from above by the natural debt limit $\bar{b}$, the finding that the variance of $\log\left(\bar{b} - b_t\right)$ is ever increasing while the average debt remains constant implies that the distribution of debt converges to a degenerate one, where a unit mass of countries are arbitrarily close to the debt limit, whereas a vanishing measure of them runs a huge surplus.

3.4 Empirical Evidence

The theoretical model presented in Section 3.2 predicts that fiscal policy in each economy is related to the relative preferences for public goods. In particular, assuming that left-leaning (right-leaning) governments are elected when voters have a stronger (weaker) taste for public goods, debt growth under left-wing governments is predicted to be lower than under right-wing governments conditional on the remaining state variables. Moreover, the conditional tax rate as well as government expenditures are predicted to be higher under left-leaning governments. If such a pattern is confirmed in the data, political shifts are indeed an important determinant of the cross-country distribution of government debt.

In this section we will provide an empirical test of this hypothesis by documenting the debt dynamics in response to political shifts in the data. One observation that motivated the work of Persson and Svensson (1989) on the strategic use of debt was that Republican administrations in the 1980s tended to accumulate more debt than Democrat ones. Here, we ask whether this is a general feature of the data, in both the U.S. and in a panel of OECD countries.[12] The empirical specification follows Bohn (1998) who has analyzed the effects of short-lived increases in U.S. government expenditures on the U.S. debt-output ratio. He finds that the debt-output ratio is mean-reverting, namely, a short-lived expenditure increase induces an increase in the debt-output ratio on impact and a subsequent reversion towards its initial level. While our data analysis also suggests mean-reversion of the debt-output ratio, here, we instead we will mostly focus on whether fiscal policy is correlated with the political inclination (left vs. right) of governments. For our analysis, we augment Bohns specification with political dummies so as to allow different debt growth first across Republican vs. Democrat administrations in the U.S., and then across governments of different ideologies in a broader set of OECD countries. The baseline empirical specification for the U.S. reads

$$\{\Delta b_t, \tau_t, g_t\} = \beta_0 + \beta_1 \text{demo}_t + \beta_2 b_t + \beta_3 u_t + \gamma' x_t + \epsilon_t, \tag{3.14}$$

where $\Delta b_t \equiv b_{t+1} - b_t$ denotes the annual change in the debt-GDP ratio, τ_t and g_t is tax revenue and expenditures as a percentage of GDP, demo_t is a dummy variable which equals one or zero when the President of the U.S. is a Democrat or a Republican, u_t stands for the

[12] In Appendix C.2 we document the data and sample selection in detail.

Table 3.1: Regression for the United States, 1956-2010

Dep. Variable	Debt Growth, Δb_t			Taxes, τ_t			Expenditures, g_t		
	Total			Corporate	Personal	Total	Non-Defense	Defense	Total
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
demo$_t$	-.296	-1.597*	-1.203**	.227**	.324	.571	.748***	-.871***	-.123
	(-.20)	(-1.87)	(-2.39)	(2.17)	(1.09)	(1.69)	(3.27)	(-3.20)	(-.31)
b_t	-.027	-.118**	-.085**	.038***	-.014	-.013	-.107***	.049**	-.058**
	(-.35)	(-2.63)	(-2.36)	(9.17)	(-.80)	(-.75)	(-8.39)	(2.25)	(-2.56)
u_t			.986***	-.165***	-.099	-.174	.515***	.164	.679***
			(5.15)	(-3.10)	(-.80)	(-1.26)	(6.21)	(1.53)	(5.11)
demo$_t$ × rec$_t$		12.237***	8.348***	-.207	-1.703***	-2.906***	.803*	.924*	1.727***
		(9.84)	(9.62)	(-1.36)	(-3.14)	(-4.73)	(1.96)	(1.80)	(3.00)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	55	55	55	55	55	55	55	55	55
Adj. R^2	0.168	0.658	0.823	0.905	0.265	0.361	0.973	0.879	0.768

Note: b_t is the-beginning-of year t debt-GDP ratio reported in the historical tables of the Office of Management and Budget, $\Delta b_t \equiv b_{t+1} - b_t$ denotes debt growth in year t. demo$_t$ is a dummy variable which equals one or zero when the President of the U.S. is a Democrat or a Republican, respectively. u_t stands for the average unemployment rate and rec$_t$ indicates years of the great recession. Control variables are the fraction of population aged over 65 and below 15. All controls are measured as yearly averages. Standard errors are adjusted for clusters in electoral term, the corresponding t-statistics are reported in parenthesis. ***, **, and * is significant at the 1%, 5%, and 10% level, respectively.

Table 3.2: Regression for the United States, 1956-2010: lagged specification

Dep. Variable	Debt Growth, Δb_{t+1}			Taxes, τ_{t+1}			Expenditures, g_{t+1}		
	Total			Corporate	Personal	Total	Non-Defense	Defense	Total
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
demo_t	-2.063*	-2.617***	-2.443***	.230**	.813**	1.156***	.337	-.738**	-.401
	(-2.12)	(-3.21)	(-3.39)	(2.43)	(2.84)	(3.51)	(1.01)	(-2.43)	(-.87)
b_t	-.076	-.090*	-.074*	.038***	.006	.008	-.121***	.056*	-.066**
	(-1.57)	(-2.00)	(-1.95)	(5.41)	(.41)	(.52)	(-7.48)	(1.82)	(-2.51)
u_t			.433**	-.130**	-.121	-.202**	.367***	.141	.508***
			(2.40)	(-2.38)	(-1.36)	(-2.25)	(3.79)	(1.03)	(3.82)
$\text{demo}_t \times \text{rec}_t$		9.296***	7.773***	-.005	-2.173***	-3.225***	.394	1.251**	1.645***
		(9.65)	(12.26)	(-.03)	(-6.11)	(-9.09)	(.89)	(2.33)	(4.34)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	54	54	54	54	54	54	54	54	54
Adj. R^2	0.270	0.434	0.457	0.884	0.361	0.395	0.937	0.848	0.616

Note: b_t is the-beginning-of year t debt-GDP ratio reported in the historical tables of the Office of Management and Budget, $\Delta b_{t+1} \equiv b_{t+2} - b_{t+1}$ denotes debt growth in year $t+1$. demo_t is a dummy variable which equals one or zero when the President of the U.S. is a Democrat or a Republican, respectively. u_t stands for the average unemployment rate and rec_t indicates years of the great recession. Control variables are the fraction of population aged over 65 and below 15. All controls are measured as yearly averages. Standard errors are adjusted for clusters in electoral term, the corresponding t-statistics are reported in parenthesis. ***, **, and * is significant at the 1%, 5%, and 10% level, respectively.

unemployment rate, and x_t is a vector of demographic controls. According to the findings of Bohn (1998), the coefficient on the current debt-output ratio, β_2, should be negative in the regression on debt growth in order to imply mean reversion of the debt-output ratio. Our coefficient of main interest however will be β_1, which measures the impact of a Democrat U.S. President on fiscal policy relative to a Republican President, while the unemployment rate will mainly serve as a proxy to control for the business cycle.

It is not obvious that fiscal policies become operative contemporaneously, therefore we will also consider the lagged specification

$$\{\Delta b_{t+1}, \tau_{t+1}, g_{t+1}\} = \beta_0 + \beta_1 \text{demo}_t + \beta_2 b_t + \beta_3 u_t + \gamma' x_t + \epsilon_{t+1}, \tag{3.15}$$

for the analysis of the U.S. where we know exactly that the stock of government debt is reported for the beginning of the year (or, similarly, for the end of the last year). Thus, the specification in Equation (3.15) implies that policies become effective with a time lag of one year. Note that in this specification the coefficient β_1 can no longer be used to test mean reversion of the debt-output ratio because b_t is now lagged two periods relative to Δb_{t+1}. In the broader set of OECD countries the exact timing of reported fiscal policies is not known and likely to be heterogeneous. In the worst case when the debt level is reported for the end of the year, we would then look a two period lagged specification which we do not consider to be appropriate. Thus, we will stick to the contemporaneous specification for the panel regressions on the cross-country sample of OECD countries.

The results for the contemporaneous specification in Equation (3.14) are reported in Table 3.1. In the left panel we find that the growth in the debt-output ratio is significantly correlated with the party in power in the U.S. over the postwar period.[13] Namely, we find that the growth in the debt-output ratio is significantly lower under a Democrat than under a Republican government. Moreover, the estimated magnitudes are big: in our most preferred specification of column (3) the debt-GDP ratio is increasing by 1.2 percentage points more per year under a Republican president compared to a Democrat president. Implying that an infinite sequence of Republican administrations would yield a substantially higher stationary debt-output ratio than one of Democrat administrations. Our findings are robust to including periods of the great recession, however, addressing the fact that debt policy in the great recession might differ from the debt policy in normal times we add an interaction term with the the political measure for the years 2008, 2009 and 2010 in columns (2) and (3) of Table 3.1. Restricting the sample to periods before the great recession (not reported) would yield very similar results as reported in column (3).

[13] In the postwar period the identification of Republicans with "right-wing" and that of Democrats with "left-wing" is not controversial, while this becomes more contentious in earlier periods.

Furthermore, in columns (2) and (3) we find that the coefficient on the initial debt level is significantly negative which confirms the findings of Bohn (1998) that debt in the U.S. is mean-reverting in the postwar period. Using the lagged specification from Equation (3.15), the left panel of Table 3.2 reports even larger effects of the type of governments on debt growth one period later, in columns (1) to (3) the coefficient on Democrat presidents is around minus 2.1 to 2.4 percentage points and statistically significant in all specifications.

The middle panel of Table 3.1 reports the conditional correlation of federal tax revenue (as a percentage of GDP) with the political inclination of the U.S. government. We look at the composition as well as the total of tax revenue. In columns (4) to (6) we find evidence that Democrat governments raise more tax revenue conditional on the state of the economy. However, the composition of the tax revenue is important. For corporate tax revenue the coefficient on the political variable is statistically significant and positive, while the coefficients on individual income and total tax revenue are only economically significant. Once we allow for the lagged specification reported in the middle panel of Table 3.2 also coefficients on personal and total of tax revenue becomes statistically significant and increases in magnitude.

Finally, we set federal government expenditures as percentage of GDP as the dependent variable in the specification of Equation (3.14). In the right panel of Table 3.1 we show that the composition in federal defense and non-defense expenditures is important. While the effect of the political variable on total expenditures reported in column (9) is not statistically significant, we find that Democrat governments spend around 0.9 percentage points less on federal defense expenditures but more on non-defense expenditures compared to Republican governments. Thus, the predictions of our model are consistent with the empirical evidence for the U.S. as long as we think of the public good as non-defense spending of the government. This pattern also prevails in the lagged specification, although the coefficient on non-defense expenditure looses in statistical significance.

The cross-country analysis for a broader set of OECD economies is less straightforward compared to the empirical analysis for the U.S. due to the large heterogeneity in political and electoral systems across countries which is a source of measurement error. A recently updated and often cited political measure of governments' party ideology is provided in Woldendorp, Keman, and Budge (2000, 2011) who assign scores for government and parliament ranging from left-wing dominance to right-wing dominance.[14] Because their political measure is only available until 2007, we complete[15] the missing values with the classification of governments on a left-right scale taken from the World Bank Database of

[14] This dataset reference for 2011 is an update of Woldendorp, Keman, and Budge (2000) to the year 2007.

[15] The exact procedure is discussed in more detail in Appendix C.2.

Table 3.3: Panel regression for the OECD

Dep. Variable	Δb_t, Reinhart & Rogoff (2009)				Δb_t, OECD (2010a)	
Sample	1950-2010		1955-2010	1955-2007	1980-2010	1980-2007
	(1)	(2)	(3)	(4)	(5)	(6)
left$_t$	-.210	-.451	-.443	-.615**	-.695*	-.765**
	(-.70)	(-1.50)	(-1.49)	(-2.06)	(-1.91)	(-2.20)
center$_t$	-.173	-.364	-.250	-.384	.293	.094
	(-.51)	(-1.07)	(-.75)	(-1.14)	(.81)	(.27)
b_t	-.035***	-.035***	-.039***	-.042***	-.091***	-.111***
	(-3.30)	(-3.31)	(-3.59)	(-3.88)	(-4.96)	(-5.49)
u_t			.381***	.388***	.709***	.740***
			(6.71)	(6.65)	(9.87)	(9.93)
left$_t$ × rec$_t$		2.813**	2.343*		2.532*	
		(2.13)	(1.79)		(1.95)	
center$_t$ × rec$_t$		3.547	2.913		2.833	
		(1.31)	(1.12)		(1.32)	
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1170	1170	1156	1092	657	586
Adj. R^2	0.333	0.337	0.341	0.305	0.481	0.468

Note: Country and year dummies are included to control for fixed effects and time effects. b_t is the-beginning-of fiscal year t debt-GDP ratio reported in Reinhart and Rogoff (2009) and OECD (2010a). left$_t$ stands for a left-wing and center$_t$ for a coalition or a center government. u_t stands for the average unemployment rate and rec$_t$ is an indicator for the great recession. Control variables are the fraction of population aged over 65 and below 15. All controls are measured as yearly averages. Robust t-statistics are reported in parenthesis. ***, **, and * is significant at the 1%, 5%, and 10% level, respectively.

Political Institutions (DPI, see Beck et al. 2001).

In Table 3.3 we report that a similar conclusion on debt growth that we found for the U.S. holds for a broader set of OECD countries. We find that in a panel regression with country and time dummies debt accumulation is less pronounced under left-wing than under right-wing governments and we find that government debt is mean reverting. Our findings for the OECD countries are robust across different datasets on debt policy[16] and sample periods including the great recession. However - not very surprisingly given the measurement error potential of the political measure - the estimated difference between right and left-wing governments is quantitatively smaller and of lower significance than reported for the U.S. in Table 3.1.[17]

Table 3.3 shows that the coefficient on the dummy for left-wing governments (with right-wing governments as the base group) is consistently negative across all specifications. Columns (1) to (4) of Table 3.3 report the results for the sample where the debt-output ratio is taken from Reinhart and Rogoff (2011). This sample spans a maximum period of 60 years for 24 OECD countries[18] Columns (1) to (3) cover the whole postwar period including the great recession and suggest that left-wing as well as centered governments accumulate less debt than right-wing governments. The fact that the coefficients remain below the statistical significance for the full sample is mainly driven by the years of the great recession. As is reported in column (4) the coefficient on left-wing governments becomes significant once we drop the recession period.

Since the debt-output ratios reported in the database of Reinhart and Rogoff (2009) are compiled from several sources and consist of a mix of gross and net central government debt as well as gross general government debt, we repeat the analysis for a shorter sample from the OECD (2010a) which consistently reports gross central government debt for all OECD member countries.[19] Columns (5) and (6) confirm our results from the previous analysis

[16] We consider on the one hand the dataset of Reinhart and Rogoff (2009) who report central and general government debt-output ratios for 27 OECD countries over the postwar period, but also the central government debt-output ratios provided in OECD (2010a) covering all OECD countries starting from 1980.

[17] There is an empirical literature focusing on the strategic use of debt driven by ideological differences across parties (see e.g. Pettersson-Lidbom, 2001). In particular, Lambertini (2003) also examines if the color of government affects the budget deficits in OECD countries but does not find significant effects. However, she uses a shorter and smaller data sample than us and does not include the current level of debt as a control variable.

[18] The countries are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Poland, Portugal, Spain, Switzerland, Sweden, United Kingdom, and the United States. We exclude Turkey from the analysis because it defaulted on its sovereign debt in 1978 and 1982 and gets a consistently low democratization score for the postwar period according to the Polity IV Project. For Iceland we drop the years of the severe financial crisis in 2008-2010.

[19] Again, Turkey as well as Iceland's 2008-2011 financial crisis are excluded from the analysis. Due to the limited availability of the political measure, this sample covers 27 OECD countries (the 24 reported for the long-run sample as well as the Czech Republic, Luxembourg, and the Slovak Republic). The details of

that left-wing government accumulate less debt compared to right-wing governments. For this sample, the coefficient on left-wing government is negative and statistically significant also if we include the years of the great recession. Overall, the empirical evidence for the OECD countries suggests that right-leaning governments increase the debt-output ratio around 0.6 to 0.8 percentage points faster than left-leaning governments conditional on the debt level at the start of the period.

In conclusion, we find that fiscal policy is indeed correlated with the political inclination of governments and that government debt is mean-reverting. In particular, we provide robust evidence that debt growth is positively correlated with right-wing parties being in government which suggest that political shifts are indeed an important determinant of the cross-country distribution of government debt. Motivated by this finding, we show in the quantitative analysis of our model in Section 3.5 that political shifts alone can generate a cross-country distribution of government debt that is a reasonable fit to the one observed in the data.

3.5 Quantitative Analysis

In this section we consider the general case of our model with an elastic labor supply and show how far shocks to the relative preference for the public good, θ, can take us in explaining the empirical cross-country distribution of government debt. As discussed in the previous section we associate the preference shock with political shifts: left-leaning (right-leaning) governments are elected when voters have a stronger (weaker) taste for public goods.[20] The Markov-perfect political equilibrium of this version of the model can no longer be solved analytically. With θ being the only idiosyncratic shock and relaxing the assumption the $\lambda = 1$, the intratemporal trade-off between private and public consumption in Equation (3.4) now becomes

$$\frac{(1-\omega)(1+\beta)}{A(T(b,\theta,R),R)} = (1 - e(T(b,\theta,R)))(1+\omega(\lambda-1))\frac{\theta}{G(b,\theta,R)},$$

where $e(\tau) \equiv -(\mathrm{d}H(\tau,R)/\mathrm{d}\tau)(\tau/H(\tau,R))$ denotes the tax elasticity of the labor supply. The generalized Euler equation reads

$$\frac{\theta}{G\left(b,\theta,R\right)} = -\frac{(1-\omega)\beta}{1+\omega(\lambda-1)}\mathrm{E}_{\theta'|\theta}\frac{\theta' G_1\left(B\left(b,\theta,R\right),\theta',R\right)}{G\left(B\left(b,\theta,R\right),\theta',R\right)},$$

the sample coverage are delegated to Table C.2 of Appendix C.2.

[20] The leftist wave of the 1960's and the neoconservative movement of the 1980's are examples of such political shifts.

and the government's budget constraint is given by

$$B(b, \theta, R) = G(b, \theta, R) + Rb - T(b, \theta, R) W(R) H(T(b, \theta, R), R).$$

Contrary to the analytical solution derived in Section 3.3, here we have to rely on a numerical analysis to study the model's implications for the stationary cross-country distribution of government debt. We use a standard projection method with Chebychev collocation[21] to approximate the functions $B(\cdot)$, $G(\cdot)$, and $T(\cdot)$ of the Markov-perfect political equilibrium. Given these policy functions, we then simulate a large number of economies over a long time-horizon until the cross-country distribution of government debt remains stationary. Finally, we repeatedly adjust the world interest rate R such that the world asset market clears in the stationary equilibrium.

3.5.1 Calibration

Given the similarities of our theories, we calibrate our model along the lines of Song, Storesletten, and Zilibotti (2012). We analyze the stationary equilibrium of our model where the world interest rate R is constant and assume that the length of a model period corresponds to 30 years. We focus on a Markov process for the political shock with two possible realizations, one for leftist governments, θ_L, and one one for rightist governments, θ_R, where $\theta_L > \theta_R$. The persistence of both shocks is symmetric and parametrized with ρ. The functional form of the household production technology reads

$$F(h, X) = \frac{\xi}{1+\xi} X \left(1 - h^{\frac{1+\xi}{\xi}}\right),$$

where $\xi > 0$ is the Frisch elasticity of labor supply. Note that the limiting case $\xi \to 0$ nests the case of inelastic labor supply that we analyzed in Section 3.3. The benchmark calibration of the model is summarized in Table 3.4. We calibrate the free parameters β, λ, X, and ξ to the certainty equivalent stationary equilibrium of the model evaluated at the average preference realization $\bar{\theta} = (\theta_L + \theta_H)/2$ and the average debt-output ratio of 0.48 that we infer from the OECD (2010a) sample for small OECD economies[22]. We target the wealth-output ratio in the world economy,[23] the tax revenue as a percentage

[21] See Judd (1998) and Miranda and Fackler (2002), for example, for details about the implementation of projection methods.

[22] When referring to the sample of small OECD countries we exclude Germany, Japan and the United States from the analysis.

[23] Since the average debt-output ratio in the data is 0.48, targeting an annualized wealth-output ratio of 3.48 sets the annualized capital-output ratio to 3. Because firms optimization behavior is competitive, the calibration implies an annualized world interest rate of around 4% which is in line with the existing quantitative macro literature (see Trabandt and Uhlig (2010), for example).

Table 3.4: Benchmark calibration

Target	Value	Parameter	Value
Capital's income share of output	1/3	α	1/3
Relative voter turnout for the old (61+) in the U.S.	25%	ω	0.25
World wealth-output ratio (annualized)	3.48	β	$(0.970)^{30}$
Average tax on labor income	34%	λ	3.73
Ratio of non-market to market earnings for young	18/33	X	1.29
Tax rate at the top of the Laffer curve	60%	ξ	2/3
Symmetric reelection probability four-year term	$\tilde{\rho}$	ρ	0.5
Debt-output ratio for high-debt OECD countries	70%	θ_L	0.216
Debt-output ratio for low-debt OECD countries	26%	θ_H	0.159

of GDP,[24] the non-market earnings to market earnings ratio for the young agents,[25] and the maximal tax rate corresponding to the top of the Laffer curve,[26] respectively. The details of this calibration are delegated to Appendix C.3. Furthermore, we parametrize the physical capital's share of output to $\alpha = 1/3$ which is standard for developed countries and the political influence of the old to $\omega = 0.25$ which is taken from Song, Storesletten, and Zilibotti (2012). For the stochastic process, we set the persistence parameter of the political shock to $\rho = 1/2$ which is consistent with the assumption that the reelection probability after a four-year term is the same for left- and right-wing governments.[27] We note that the persistence parameter ρ is an important determinant of the stationary distribution's shape and we will also report numerical results for alternative values of the persistence. A general observation is that the stationary distribution of government debt will become more polarized towards the tails of the distribution as ρ increases. Finally, the high (low) realization of the stochastic process is calibrated to match the 1980 to 2010 average debt-output ratio in small OECD economies with a debt level below (above) the median debt level, corresponding to 26% and 70%, respectively.

[24] Over the period 1980-2010, the average central government tax revenue reported in OECD (2010c) in the small OECD economies was 22.7% of GDP. With $\alpha = 1/3$ this translates into 34% of labor income.

[25] Aguiar and Hurst (2007) find that the ratio of market hours worked to total hours worked in the U.S. is 33/51. Implying that the income of non-market to market earnings is given by 18/33.

[26] As in Trabandt and Uhlig (2010), we assume that the top of the Laffer curve is at $\bar{\tau} = 60\%$.

[27] Since we assume that one period corresponds to thirty years, any moderate reelection probability $\tilde{\rho}$ over four years will imply a ρ close to 1/2. To see this, note that the reelection probability over thirty years can be approximately computed as

$$\begin{pmatrix} \rho \\ 1-\rho \end{pmatrix} \approx \begin{pmatrix} \tilde{\rho} & (1-\tilde{\rho}) \\ (1-\tilde{\rho}) & \tilde{\rho} \end{pmatrix}^7 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \approx \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}.$$

3.5.2 Cross-Country Distribution of Government Debt

We draw the stationary cross-country distribution of government debt from a simulation with one million small open economies over two hundred years. In the data however, we only observe the debt-output ratios of a small number of OECD economies over a limited time period.[28] Therefore, in Figure 3.1 we rely on the estimated kernel densities[29] to compare the empirical and the simulated distribution of the model. The comparison of

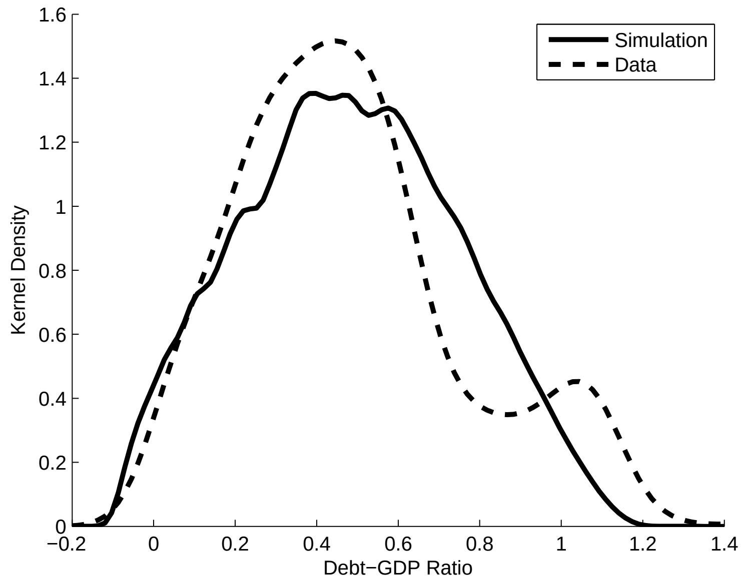

Figure 3.1: Comparison: empirical and simulated distribution.

the two distributions reveals that even though we only target two conditional means of the empirical distribution, the stationary distribution generated by the model is a reasonable fit to the distribution we observe in the data.[30] Not very surprisingly, the model has a hard time explaining the upper tail of the empirical distribution which suggests even a bimodal distribution. However, the simulated distribution has at least more mass in the right tail as the empirical pattern would also suggest.

While the good fit of the stationary distribution is robust to variation in the remaining parameters, it is sensitive to the calibration of the political shock's persistence, ρ. In Figure 3.2, we report the resulting distribution of government debt for a range of alternative

[28] The empirical distribution is based on the central government debt to output ratios reported in OECD (2010a). We consider the same set of countries as in the empirical analysis of of Section 3.4, excluding Germany, Japan, and the United States.

[29] We use an Epanechnikov kernel with optimal bandwidth for both distributions in separate.

[30] In principle, our model captures *net* central government debt which includes the assets and not only the liabilities of the central government. However, the United Kingdom is the only OECD country that reports such a debt measure in the Reinhart and Rogoff (2009) dataset. We therefore use *gross* central government debt as an empirical proxy instead.

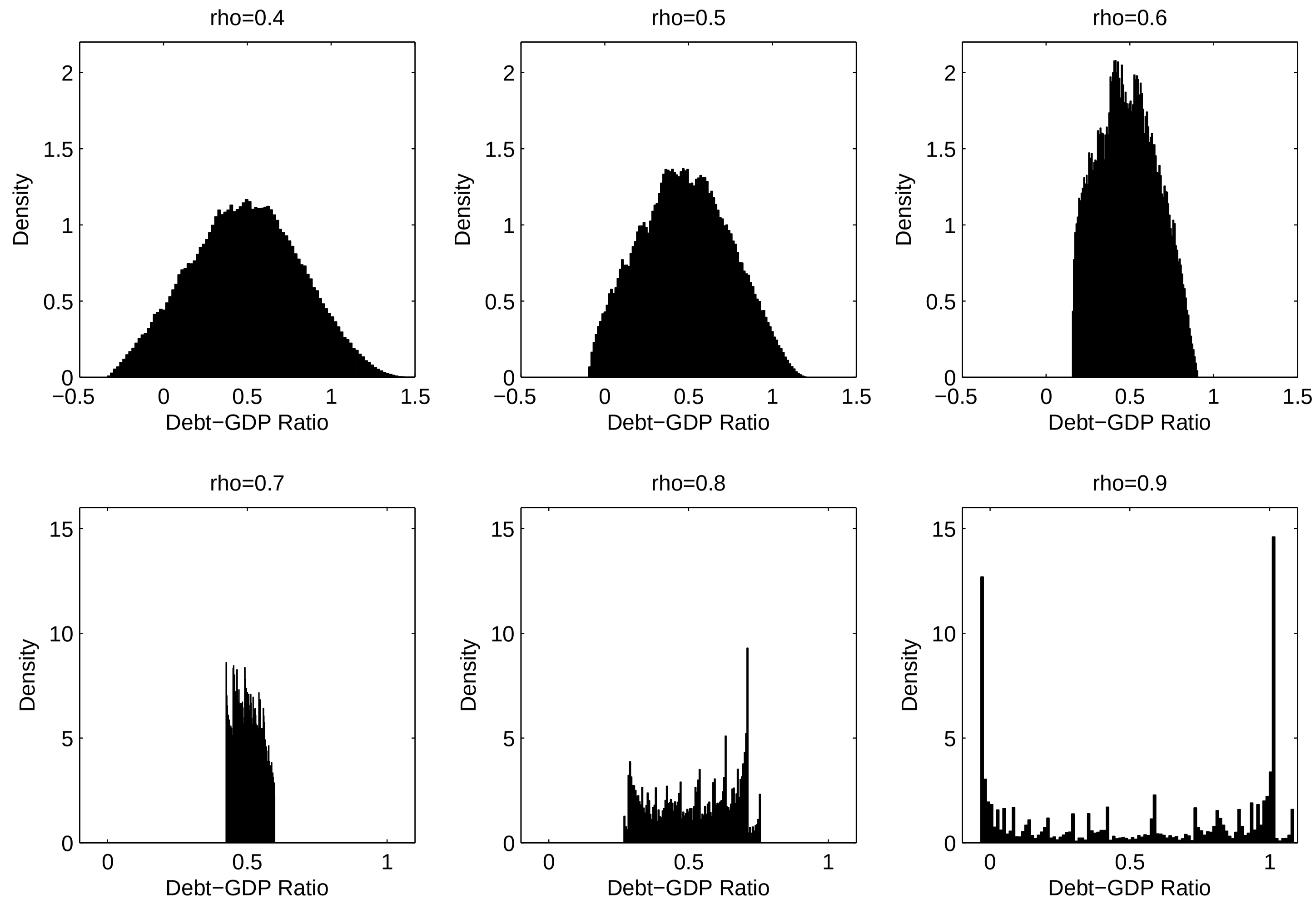

Figure 3.2: Simulation of the cross-country distribution of government debt.

calibrations of the persistence parameter. The middle panel of the top row shows the histogram for the benchmark calibration listed in Table 3.4. The panels surrounding the benchmark simulation illustrate the distribution of government debt for alternative values of the persistence parameter ρ, keeping the remaining parameters including the values for θ_L and θ_H as well as the world interest rate at the benchmark values. The panels show that the distribution of government debt becomes more polarized as the persistence of the political shock increases. In the limiting case, $\rho \to 1$ (which is left out in Figure 3.2), the distribution is fully polarized with two mass points at each end of the distribution, for example. For values of ρ close to 0.4 the distribution of government debt is close to normally distributed, while the distribution becomes more right-skewed for higher values of the persistence reported in the top row of Figure 3.2.

Overall, the simulations show that our model is able to deliver a reasonable fit to the empirical distribution of government debt with the two-state political shock alone, as long as the persistence parameter ρ is around 1/2. We acknowledge that other shocks, demographic or fiscal, as well as *ex ante* heterogeneity might also influence the empirical distribution of government debt. However, allowing for more states, shocks, and persistent heterogeneity would also increase the degrees of freedom in the model's calibration and we leave the explorations of other channels for future research.

3.6 Conclusions

In this paper we have proposed a positive theory of cross-country government debt distribution. The concern of young voters for future public good provision offsets the desire to pass the bill to future generations and to accumulate excessive government debt. While our model is flexible enough to capture different types of idiosyncratic shocks that might influence fiscal policy in each of the economically integrated economies, we provide evidence that political shifts are of particular interest. In times where people have a high preference for public goods relative to the private good (or, as we interpret it, when a leftist government comes to power) debt accumulation is less pronounced than in times where people have a lower preference for relative public goods. Alternatively, the model can also be interpreted as a standard rich-poor redistributive conflict. In times where the poor have a stronger influence on the political process (leftist periods) governments accumulate less debt than when the rich have a tighter control on political power (rightist periods). We document empirical support for this prediction in a panel of OECD countries and in separate for the U.S. time series.

Our theory provides a reasonable fit the empirical distribution of government debt with a parsimonious calibration of the stochastic process for the political shock. In line with the empirical pattern of debt-output ratios, our theory also predicts a stationary distribution of government debt, it can match the high variation in the debt-output ratio across countries by allowing for variation in the preference for public goods, and it allows countries to move within the stationary distribution over time.

Our analysis is subject to a number of caveats. Our model does not deal with the determination of government debt under coalition governments. While for the U.S. the assumption of left- and right-wing governments is less disputable, for OECD countries the consideration of coalition governments might be another important dimension determining fiscal policy. Moreover, permanent cross differences-country differences for public goods might complement the determination of government debt distribution. For example, differences in the efficiency of public good provision may affect voters' preferences for public savings. It is often argued that Italy, a country with one of the largest public debts, has an inefficient public administration, while the public sector is more efficient in Scandinavian countries which have a lower propensity to indebtedness. We have maintained throughout the assumption that governments are committed to repay their debt. The analysis could be enriched by endogenizing the incentive of governments to repay debt. Integrating our analysis with the insights of the sovereign debt literature may give rise to novel insights but requires non-trivial extensions which are left to further research.

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Appendix C

A Theory of Cross-Country Government Debt Distribution

C.1 Additional Tables

Table C.1: Mobility in the cross-country distribution of government debt

	AUS	CHL	CHE	NOR	DEU	IRL	ESP	FIN	DNK	SWE
1950s	10	1	6	5	1	8	7	3	7	6
1960s	8	3	2	7	1	10	6	4	2	6
1970s	7	5	1	8	2	10	3	1	2	6
1980s	2	8	1	4	3	10	3	1	9	8
1990s	1	3	1	3	2	8	4	5	8	7
2000s	1	1	2	2	3	3	4	4	5	5
	CAN	TUR	AUT	FRA	PRT	USA	BEL	GRC	JPN	
1950s	9	4	3	5	4	9	8	2	2	
1960s	9	7	3	5	4	8	9	5	1	
1970s	9	7	4	4	3	8	9	6	5	
1980s	7	4	5	5	2	6	9	6	7	
1990s	9	2	6	5	4	6	10	9	7	
2000s	6	6	7	7	8	8	9	9	10	

Note: Categorizes each country according to the decile in the cross-country distribution of the average gross central government debt to output ratio over the decades of the postwar period. The data is taken from Reinhart and Rogoff (2009).

C.2 Data

This appendix describes the data and sample selection used in the empirical analysis of Section 3.4 in more detail.

In the time series regressions reported in Table 3.1 we consider time series data on U.S. fiscal policy from the historical tables of the U.S. Office of Management and Budget. In the panel regression reported in Table 3.3 we rely on the one hand on the debt-output ratios provided in Reinhart and Rogoff (2009)[1] for the whole postwar period in 24 OECD countries, as well as the central governmen debt to ouptut ratios from the OECD (2010a) National Account Statistics available from 1980 onwards for 27 OECD countries. The data for unemployment and the demographic structure are taken from the OECD (2010b) Employment and Labour Market Statistics with the earliest observations available from 1950. The country-specific sample coverage for the panel regressions specified in columns (3) and (5) of Table 3.3 is reported in Table C.2. We do neither interpolate nor extrapolate any of the data points, the data enter the regressions as reported in the original sources mentioned previously. Next to the coverage of time periods, the main difference between the two samples is that the latter additionally covers Luxembourg as well as the Czech and the Slovak Republic.

The most challenging data issue concerns measuring the governments' ideology across countries and over time. We avoid the problem of cross-country comparability between governments political ideologies by including country-specific fixed effects in the regressions. Our preferred political measure is from Woldendorp, Keman and Budge (2011), henceforth labeled $\mathrm{pol}_{\mathrm{WKB}}$, which assign scores for government and parliament ranging from 1 ("right-wing dominance") to 5 ("left-wing dominance"). The criterion for "dominance" is set by the share of seats in government and parliament. Since $\mathrm{pol}_{\mathrm{WKB}}$ is only available until 2007, we complete more recent years using the Database of Political Institutions (DPI) provided by the World Bank. The latter political measure is based on the classification of the political inclination of the chief executive's party as suggested in Beck et al. (2001) and has three indicators that stand for right, center, and left governments, respectively. To be consistent with the World Bank measure, we reduced $\mathrm{pol}_{\mathrm{WKB}}$ to a three-level indicator: -1 for right ($\mathrm{pol}_{\mathrm{WKB}} \leq 2$), 0 for center ($2 < \mathrm{pol}_{\mathrm{WKB}} < 4$) and 1 for left ($\mathrm{pol}_{\mathrm{WKB}} \geq 4$) dominated governments.

[1] For Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Japan, Norway, Portugal, Spain, Switzerland, Sweden, and the U.S. they report gross central government debt. For the United Kingdom only net central government debt is available, and the times series of Iceland, Italy, Netherlands, New Zealand, and Poland only cover gross general government debt.

Table C.2: Panel regression: sample coverage

Sample	AUS	AUT	BEL	CAN	CHE	CZE	DEU	DNK	ESP
Reinhart & Rogoff (2009)	1956-2010	1956-2010	1956-2010	1956-2010	1955-2007	-	1956-2010	1956-2010	1978-2010
OECD (2010a)	1980-2010	1980-2010	1980-2010	1981-2010	1986-2007	1993-2007	1980-2010	1980-2010	1980-2010
	FIN	FRA	GBR	GRC	HUN	IRL	ISL	ITA	JPN
	1956-2010	1956-2010	1956-2010	1956-2010	1992-2010	1956-2010	1961-2007	1959-2010	1955-2009
	1990-2010	1992-2010	1998-2010	1993-2010	1992-2010	1981-2010	1980-2007	1980-2010	1980-2010
	LUX	NLD	NOR	NZL	POL	PRT	SVK	SWE	USA
	-	1956-2010	1956-2010	1956-2010	1992-2009	1977-2010	-	1956-2010	1956-2010
	1990-2010	1980-2010	1982-2010	1992-2010	1992-2010	1981-2010	1994-2010	1980-2010	1980-2010

C.3 Calibration

This appendix describes some details of the model's calibration in the quantitative analysis of Section 3.5. We will focus here on the calibration of the parameters ξ, X, β, λ that we match to non-trivial moments of the model's certainty equivalent steady-state.

(1) The home production function implies that the labor supply is given by

$$H(\tau, R) = \arg\max_h (1-\tau)W(R)h + F(h) + F(0)/R = \left[\frac{(1-\tau)W(R)}{X}\right]^{\xi}.$$

The tax rate corresponding to the top of the Laffer curve then solves

$$\bar{\tau} = \arg\max_{\tau} \tau W(R)H(\tau, R) = \frac{1}{1+\xi},$$

and that the Frisch elasticity of labor supply, $\xi = (1-\bar{\tau})/\bar{\tau}$, is determined by targeting $\bar{\tau}$.

(2) The non-market earnings to market earnings ratio is given by

$$\begin{aligned}\frac{\text{NME}}{\text{ME}} &= \frac{F(H(\tau, R))}{W(R)H(\tau, R)} = \frac{\frac{\xi}{1+\xi}X\left(1 - [(1-\tau)W(R)]^{1+\xi}X^{-(1+\xi)}\right)}{W(R)[(1-\tau)W(R)]^{\xi}X^{-\xi}} \\ &= \frac{\frac{\xi}{1+\xi}\left(X^{1+\xi} - [(1-\tau)W(R)]^{1+\xi}\right)}{W(R)[(1-\tau)W(R)]^{\xi}}.\end{aligned}$$

Thus, the productivity parameter of home production X is pinned down by targeting NME/ME,

$$X = \left(\frac{\text{NME}}{\text{ME}}\frac{1+\xi}{\xi}W(R)[(1-\tau)W(R)]^{\xi} + [(1-\tau)W(R)]^{1+\xi}\right)^{\frac{1}{1+\xi}}.$$

(3) The international capital market clearing condition implies that world wealth must be equal to world savings. Since in the certainty equivalent steady-state all countries are of the same type, this implies

$$\begin{aligned}\frac{b+K}{Y} &= \frac{S(\tau, R)}{Y(\tau, R)} = \frac{\frac{\beta}{1+\beta}[(1-\tau)W(R)H(\tau, R) + F(H(\tau, R))] - \frac{1}{1+\beta}F(0)/R}{(1-\alpha)^{-1}W(R)H(\tau, R)} \\ &= \frac{(1-\alpha)\beta}{1+\beta}\left(1 - \tau + \frac{\text{NME}}{\text{ME}}\right) - \frac{1-\alpha}{1+\beta}\frac{F(0)/R}{W(R)H(\tau, R)}.\end{aligned}$$

Targeting the world wealth-output ratio, $(b+K)/Y$, and the labor income tax rate,

τ, pins down the subjective discount factor β,

$$\beta = \frac{(1-\alpha)\frac{F(0)/R}{W(R)H(\tau,R)} + \frac{b+K}{Y}}{(1-\alpha)(1-\tau+\frac{\text{NME}}{\text{ME}}) - \frac{b+K}{Y}}.$$

Because one period in the model corresponds to 30 years, the annualized discount factor is derived from $\beta_A = \beta^{1/30}$.

(4) Finally, in the certainty equivalent steady-state the static optimality conditions of the Markov-perfect political equilibrium imply that

$$\lambda = \frac{1-\omega}{\omega}\left[\frac{(1+\beta)g}{A(\tau,R)(1-e(\tau))\bar{\theta}} - 1\right],$$

where

$$g = (1-R)b + \tau W(R)H(\tau,R) = W(R)H(\tau,R)\left[\frac{b}{Y}\frac{1-R}{1-\alpha} + \tau\right].$$

Thus targeting τ pins also down λ conditional on the certainty equivalent preference for the public good, $\bar{\theta} = (\theta_L + \theta_R)/2$. Note that $\bar{\theta}$ is chosen to target b/Y in the data. This completes the discussion of the model's calibration.